Order Optimal Information Spreading Using Algebraic Gossip

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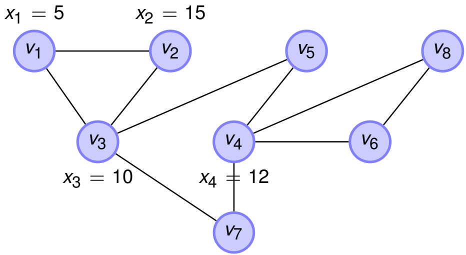
PODC 2011

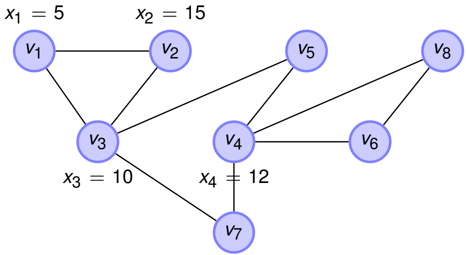
The k-Dissemination Problem



all nodes need all the k values

as quickly as possible

The k-Dissemination Problem



all nodes need all the k values

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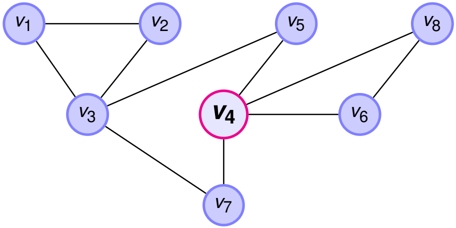
Uniform
Algebraic
Gossip

Tree Based
Algebraic
Gossip

Gossip Algorithm

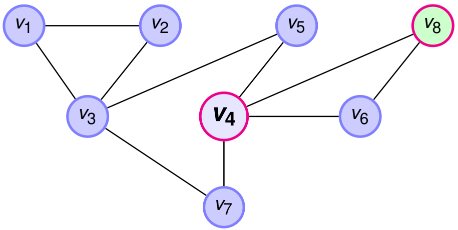
what is algebraic gossip?

Gossip Algorithm



Time is measured in **rounds**

Gossip Algorithm

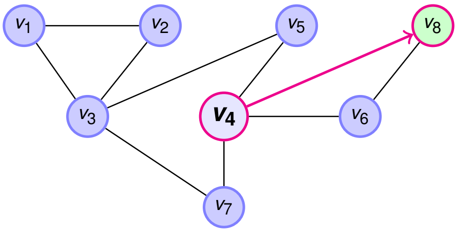


Time is measured in **rounds**

choose a partner

uniformly	non uniformly

Gossip Algorithm

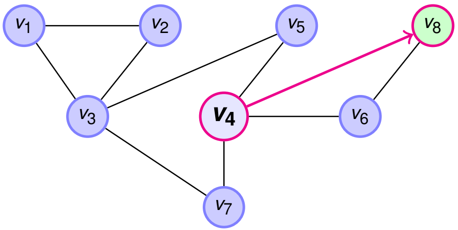


Time is measured in **rounds**

choose a partner	
uniformly	non uniformly

send a message		
push	pull	exch.

Gossip Algorithm

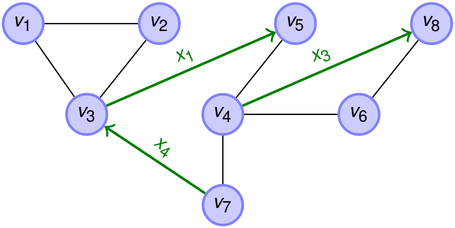


Time is measured in **rounds**

choose a partner		send a message			message content
uniformly	non uniformly	push	pull	exch.	?

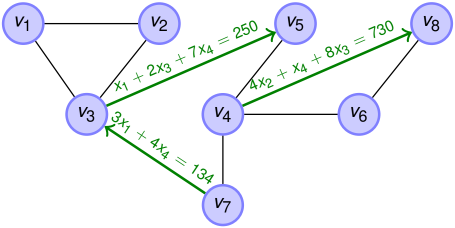
Algebraic Gossip

instead of sending randomly chosen values...



every message – a single value

nodes send random linear combinations



every message – linear equation

Algebraic Gossip

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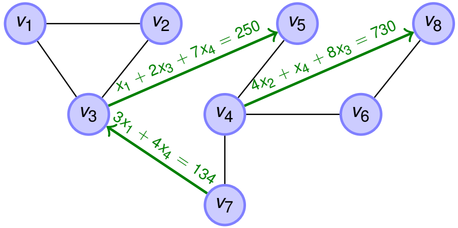

every message – linear equation

nodes store equations in a matrix form:

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{2} & \mathbf{7} \\ 2 & 0 & 0 & 7 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \mathbf{250} \\ 45 \\ 78 \\ 0 \end{bmatrix}$$

Algebraic Gossip

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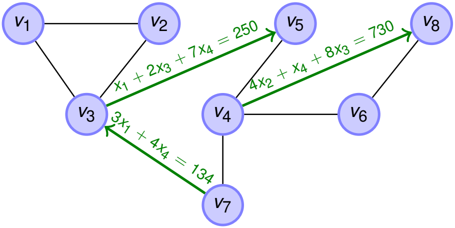

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random coefficients $\in \mathbf{F_q}$

Algebraic Gossip

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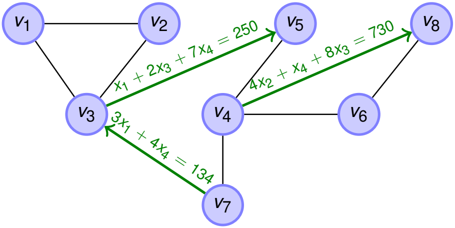

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random coefficients $\in \mathbf{F_q}$

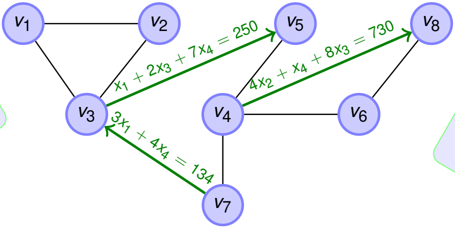
$$\mathbf{1} \times \quad 1x_1 + 0x_2 + 2x_3 + 7x_4 = 250$$

$$\mathbf{2} \times \quad 2x_1 + 0x_2 + 0x_3 + 7x_4 = 45$$

$$5x_1 + 0x_2 + 2x_3 + 21x_4 = 340$$

Algebraic Gossip

nodes send random linear combinations



rank *k* – node finishes

only **helpful** messages are stored

every message – linear equation

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$$\begin{matrix}\mathbf{1}\\\mathbf{2}\\\mathbf{0}\\ \\\end{matrix}\begin{bmatrix}1&0&2&7\\2&0&0&7\\1&1&0&0\\0&0&0&0\end{bmatrix}\cdot\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}250\\45\\78\\0\end{bmatrix}$$

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So, Why Algebraic Gossip is Faster?

Without Algebraic Gossip (Random Message Selection)

$$\Pr(\textit{helpful}) = \frac{1}{k}$$

$[x_1, x_2, x_3, \ldots x_k]$ A → B $[x_1, x_2{=}?, x_3, \ldots x_k]$

A has all values

A sends a **random message**

B is missing one value

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With Algebraic Gossip (Random Linear Equations)

$\begin{bmatrix} \text{rank } k \end{bmatrix}$ — A —— $\Pr(\textit{helpful}) = 1 - \frac{1}{q}$ ——→ B — $\begin{bmatrix} \text{rank } k-1 \end{bmatrix}$

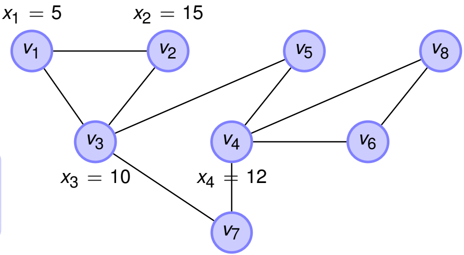
k indep. equations

A sends a **random linear equation**

$k-1$ indep. equations

$$\frac{q^k - q^{k-1}}{q^k} = 1 - \frac{1}{q}$$

Research Goal



***k*-Dissemination problem**

Research Goal

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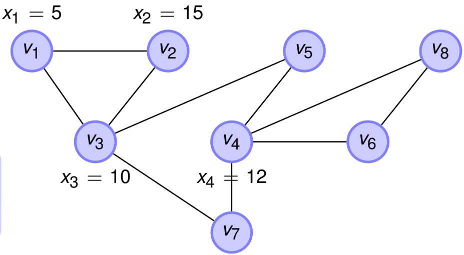

Analyze **uniform** algebraic gossip

- is it optimal?
- for which graphs?

Study **non-uniform** algebraic gossip

- we propose **tree based** algebraic gossip

Related Work

k-Dissemination with **uniform** algebraic gossip

- [Deb et al., 2006] – **complete graph**
  - For $k \gg \ln^3 n$: **Tight bound** – $\Theta(k)$.

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  - **Upper bound** – $O(\Delta n)$ for **any graph.**
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- [Haeupler, 2010] – **any graph**
  - For $k = \Omega(n)$: **Tight bound** – $\Theta(n/\gamma)$.
  - For $k < o(n)$: **Not tight for: line, grid, binary tree, ...**

Results

two main results

1st Result

k-Dissemination

Uniform algebraic gossip

1st Result

Any graph

$O(\Delta(k + \log n + D))$

max degree

diameter

***k*-Dissemination**

Uniform algebraic gossip

1st Result

Any graph

$O(\Delta(k + \log n + D))$

max degree (Δ)

diameter (D)

Const max degree graphs

$\Theta(k + D)$

Optimal!

***k*-Dissemination**

Uniform algebraic gossip

1st Result

Any graph

$O(\Delta(k + \log n + D))$

max degree

diameter

Const max degree graphs

$\Theta(k + D)$

Optimal!

***k*-Dissemination**

Uniform algebraic gossip

Results hold for

push, pull, exch

1st Result

Any graph

$O(\Delta(k + \log n + D))$

max degree

diameter

Const max degree graphs

$\Theta(k + D)$

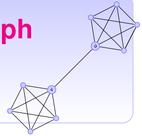
Optimal!

***k*-Dissemination**

Uniform algebraic gossip

Barbell graph

$\Omega(nk)$



Bad!

Results hold for

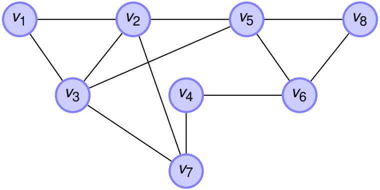
push, pull, exch

2nd Result

***k*-Dissemination**

Tree based algebraic gossip

2nd Result



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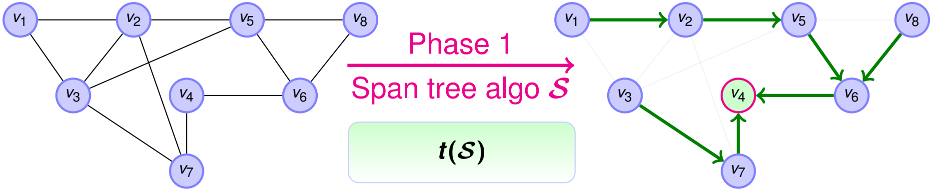

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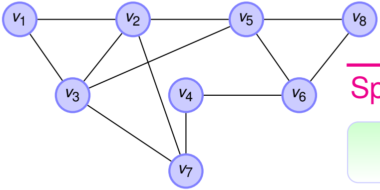
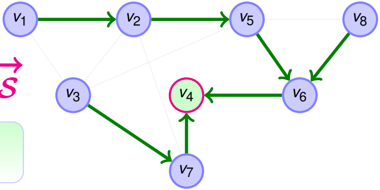


Phase 1

Span tree algo $\mathcal{S}$

$\boldsymbol{t(\mathcal{S})}$



Phase 2

k-**Dissemination**

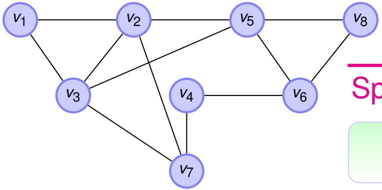
Tree based algebraic gossip

Algebraic gossip
only with parent

✗ $O(\Delta(k + \log n + D)$

✓ $O(\boldsymbol{t(\mathcal{S})} + k + \log n + \boldsymbol{d(\mathcal{S})})$

2nd Result



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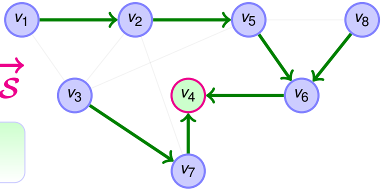

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✓ $O(\boldsymbol{t(\mathcal{S})} + k + \log n + \boldsymbol{d(\mathcal{S})})$

Any graph, $k = \Omega(n)$

$\Theta(n)$

Optimal!

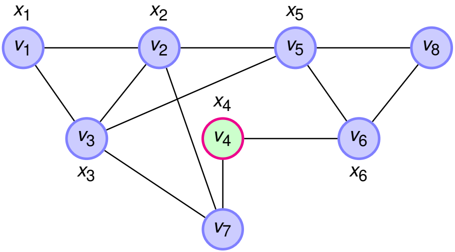
Proof Overview

k-Dissemination with **uniform** algebraic gossip

$$O(\Delta(k + \log n + D))$$

Converting a Graph to a System of Queues

initial graph with an arbitrary node v_4



when v_4 will finish the protocol?

Converting a Graph to a System of Queues

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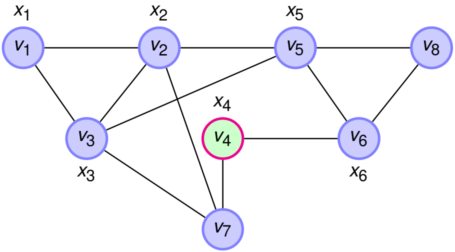

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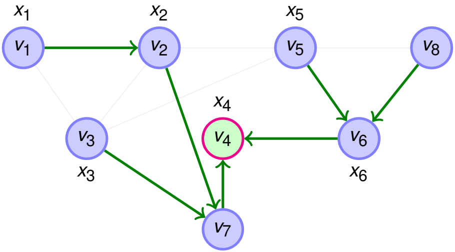
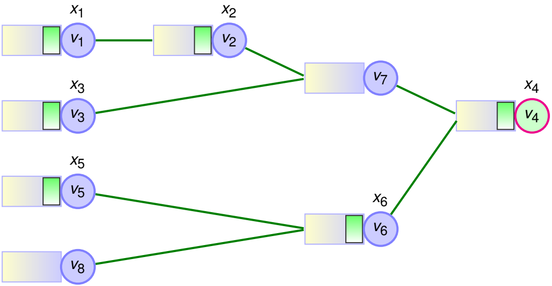
BFS spanning tree rooted at v_4



we **ignore** the messages coming from other edges

Converting a Graph to a System of Queues

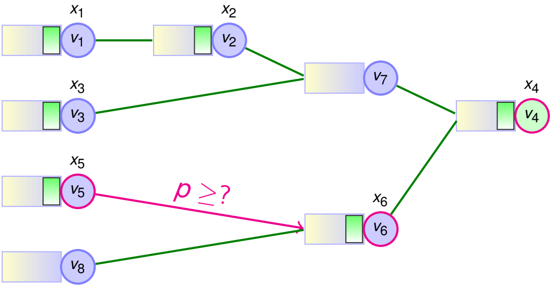


customers are **helpful** messages

customers increase node's rank

a node need k **helpful** messages to finish

Converting a Graph to a System of Queues



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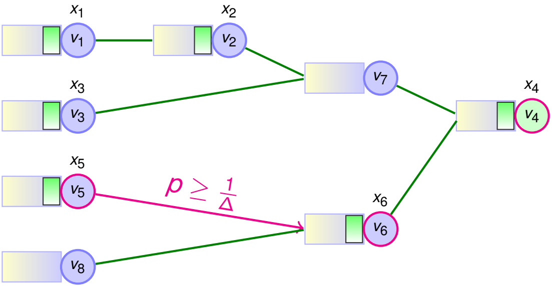

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v_5 chooses v_6 w.p. $\geq \frac{1}{\Delta}$

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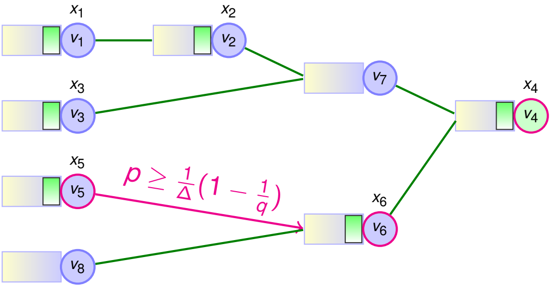

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v_5 chooses v_6 w.p. $\geq \frac{1}{\Delta}$

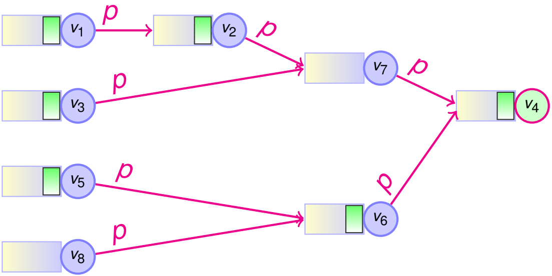
customers increase node's rank

message is **helpful** w.p. $\geq (1 - \frac{1}{q})$

a node need k **helpful** messages to finish

service time is $\sim \text{Geom}(p)$

Exponential Servers Instead of Geometric



If $X \sim \text{Geom}(p)$, and $Y \sim \text{Exp}(p)$, then: $\Pr(Y > t) \geq \Pr(X > t)$

exponential server is slower than geometric

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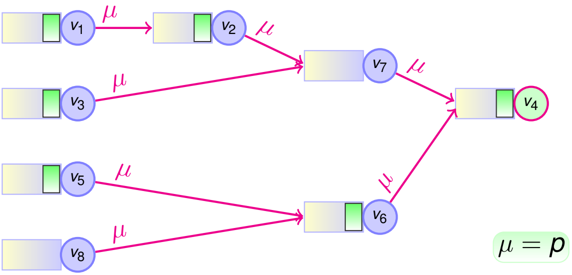

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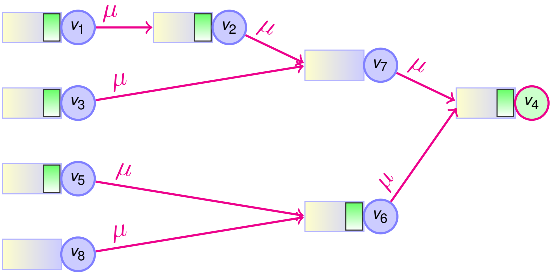
we replace servers, thus increasing the stopping time

Line is Slower Than Tree

When does the

last customer

leave the system?



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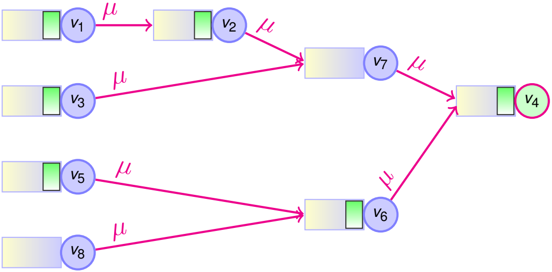
Reduce tree to line



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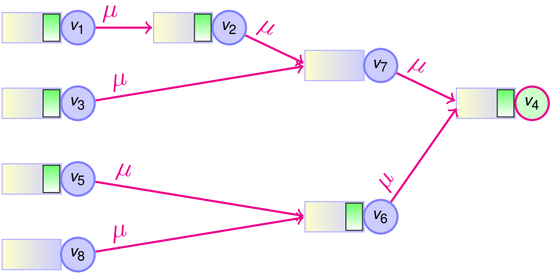

Take all customers out and use **Jackson theorem**

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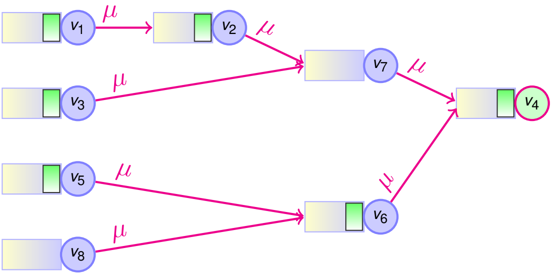

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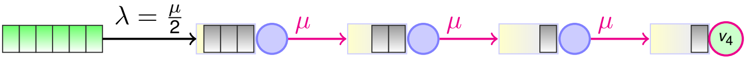

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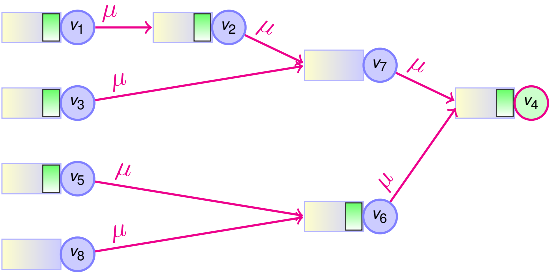

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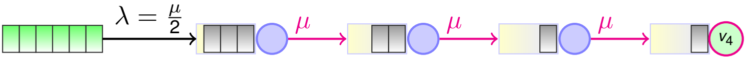

Inter-arrival time $\sim$Exp($\frac{\mu}{2}$)

All arrive after $O((k + \log n)/\mu)$

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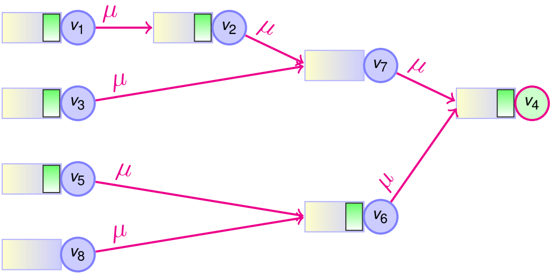

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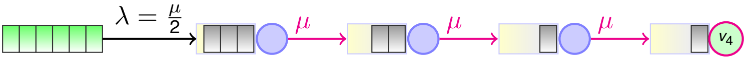

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Time to cross D queues $O((D + \log n)/\mu)$

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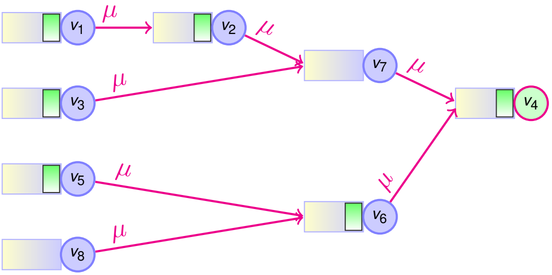

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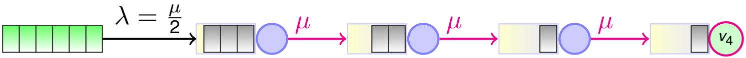

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Total: $O((k + \log n + D)/\mu) = O(\Delta(k + \log n + D))$ rounds

Summary

k-Dissemination problem

Summary

***k*-Dissemination problem**

Uniform algebraic gossip

Tree based algebraic gossip

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$O(\Delta(k + \log n + D))$

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Const max degree graphs

$\Theta(k + D)$

Optimal!

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Summary

k-Dissemination problem

Uniform algebraic gossip	**Tree based** algebraic gossip
Any graph $O(\Delta(k + \log n + D))$	**Any graph** $O(\boldsymbol{t(\mathcal{S})} + k + \log n + \boldsymbol{d(\mathcal{S})})$
Const max degree graphs $\Theta(k + D)$ **Optimal!**	**Any graph, $k = \Omega(n)$** $\Theta(n)$ **Optimal!**

Summary

k-**Dissemination problem**

Uniform algebraic gossip

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$O(\Delta(k + \log n + D))$

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Open questions

Optimal cases

What $\mathcal{S}$ to use?

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What $\mathcal{S}$ to use?

THANK YOU!

Algebraic Gossip – Overhead?

nodes store equations in a matrix form:

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{2} & \mathbf{7} \\ 2 & 0 & 0 & 4 \\ 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \mathbf{250} \\ 45 \\ 78 \\ 308 \end{bmatrix}$$

$a_i \in \mathbf{F_q}$ $\quad x_i \in \mathbf{F_q^r}$ $\quad \sum a_i x_i \in \mathbf{F_q^r}$

$$x_1 + 2x_3 + 7x_4 = 250$$

$1, 0, 2, 7$	250

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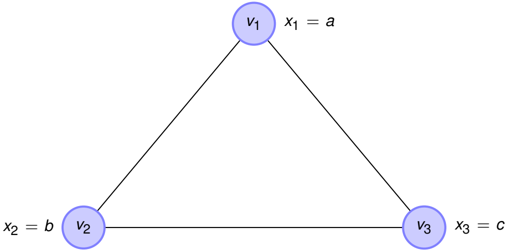
1, 0, 2, 7	250

$a_i \in \mathbf{F_q}$ $\quad \sum a_i x_i \in \mathbf{F_q^r}$

- Initial value size: $r \log q$ bits.
- Overhead (coefficients): $k \log q$ bits.
- Bits efficiency: $\frac{r \log q}{k \log q + r \log q} = \frac{r}{k+r} \to 1$.

Algebraic Gossip – Example

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}$$



$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix}$$

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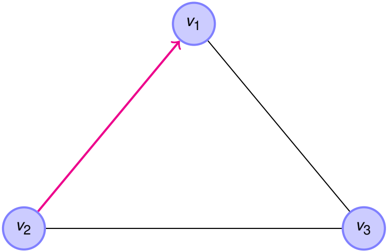

random coefficients

$$\begin{matrix} 3 \\ 2 \\ 5 \end{matrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix}$$

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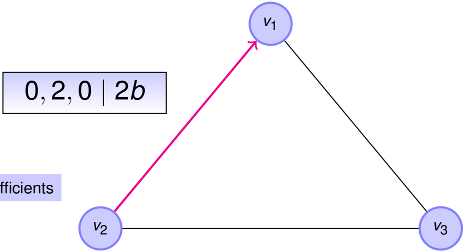

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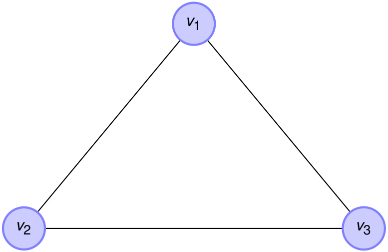
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a \\ 2b \\ 0 \end{bmatrix}$$



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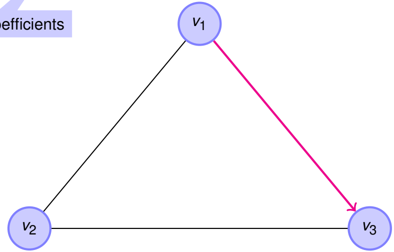
Algebraic Gossip – Example

$$\begin{matrix} 5 \\ 6 \\ 2 \end{matrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a \\ 2b \\ 0 \end{bmatrix}$$

random coefficients

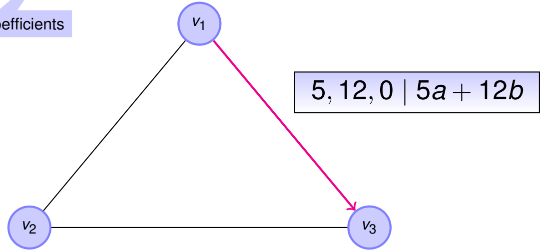


$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix}$$

Algebraic Gossip – Example

$$\begin{array}{c}5\\6\\2\end{array}\begin{bmatrix}1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 0\end{bmatrix}\cdot\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}a\\2b\\0\end{bmatrix}$$

random coefficients



$$\begin{bmatrix}0 & 0 & 0\\0 & 1 & 0\\0 & 0 & 0\end{bmatrix}\cdot\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\b\\0\end{bmatrix}$$

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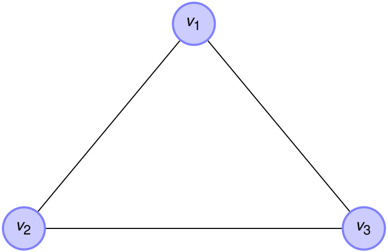

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 5 & 12 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5a + 12b \\ 0 \\ c \end{bmatrix}$$

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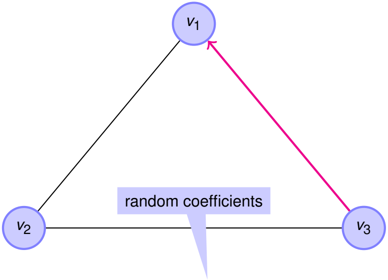

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$$\begin{matrix}4\\3\\6\end{matrix}\begin{bmatrix}5 & 12 & 0\\0 & 0 & 0\\0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}5a+12b\\0\\c\end{bmatrix}$$

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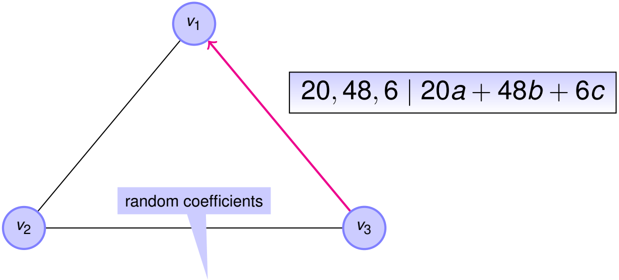

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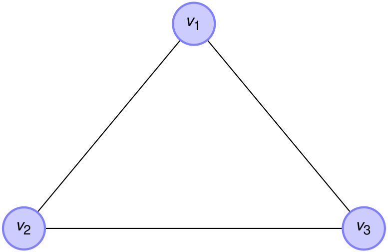
Algebraic Gossip – Example

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 20 & 48 & 6 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a \\ 2b \\ 20a + 48b + 6c \end{bmatrix} \quad \Rightarrow \quad \begin{aligned} x_1 &= a \\ x_2 &= b \\ x_3 &= c \end{aligned}$$



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