Distributed Computing on Core-Periphery Networks: Axiom-Based Design

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Networks for **Distributed Computing**

- What do we want?
  - **fast running times**, robust, small diameter, bounded degree?
  - **cost efficient**: communication links, nodes memory

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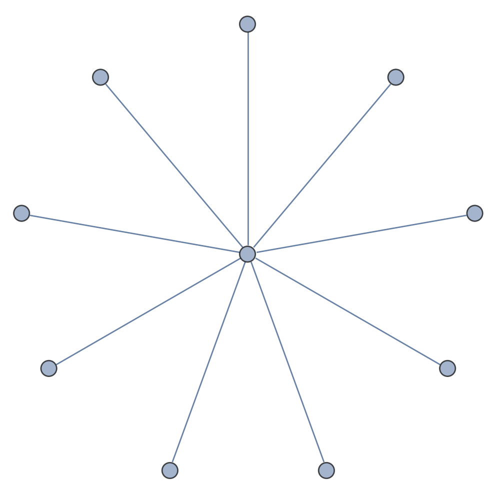
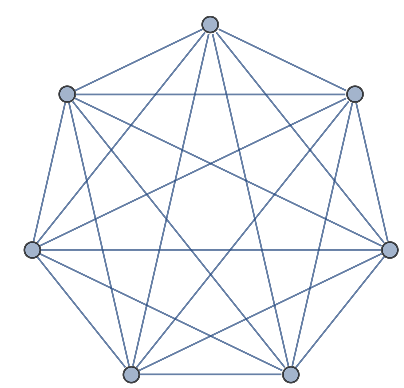
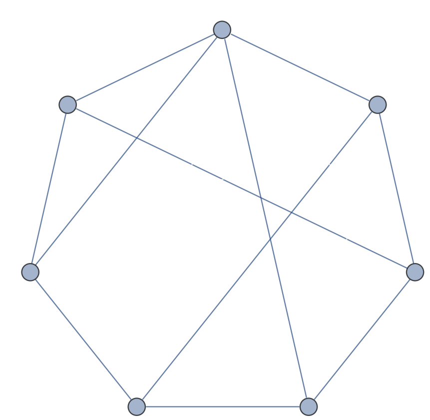
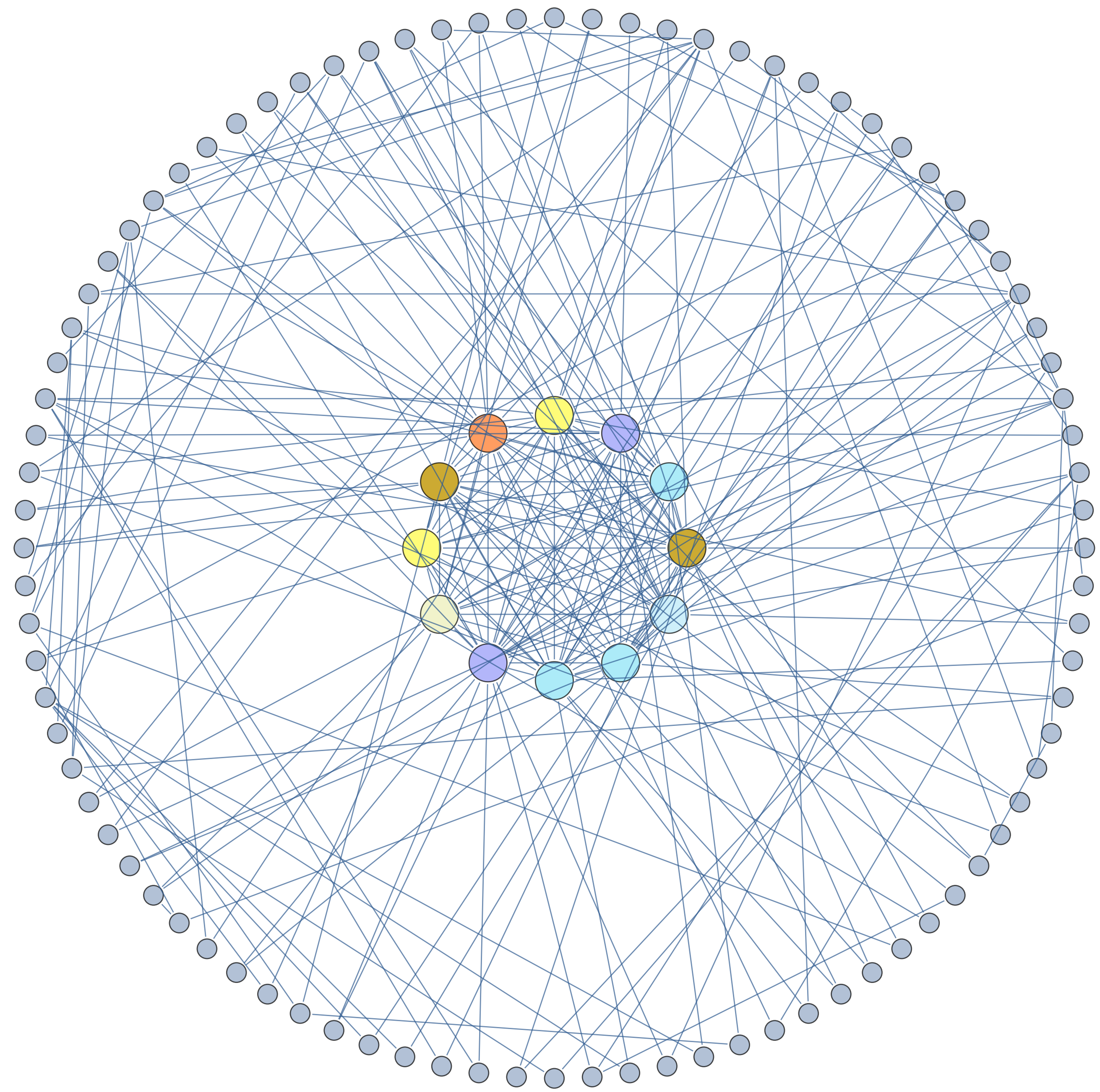
- Classic examples:
  - *Stars*
  - *Cliques*
  - *Bounded degree Expanders*





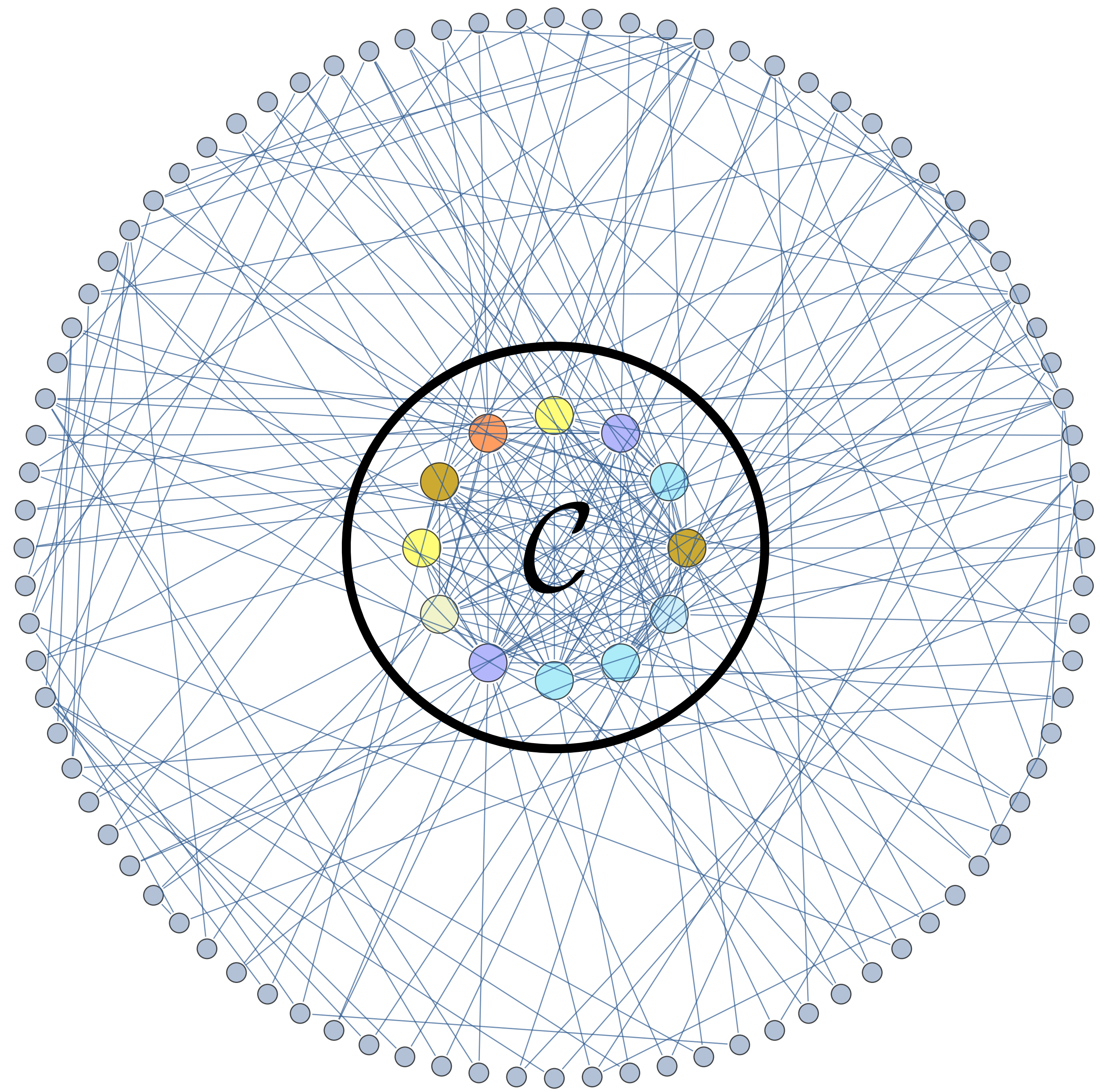


Core-Periphery Structure



Core-Periphery Structure

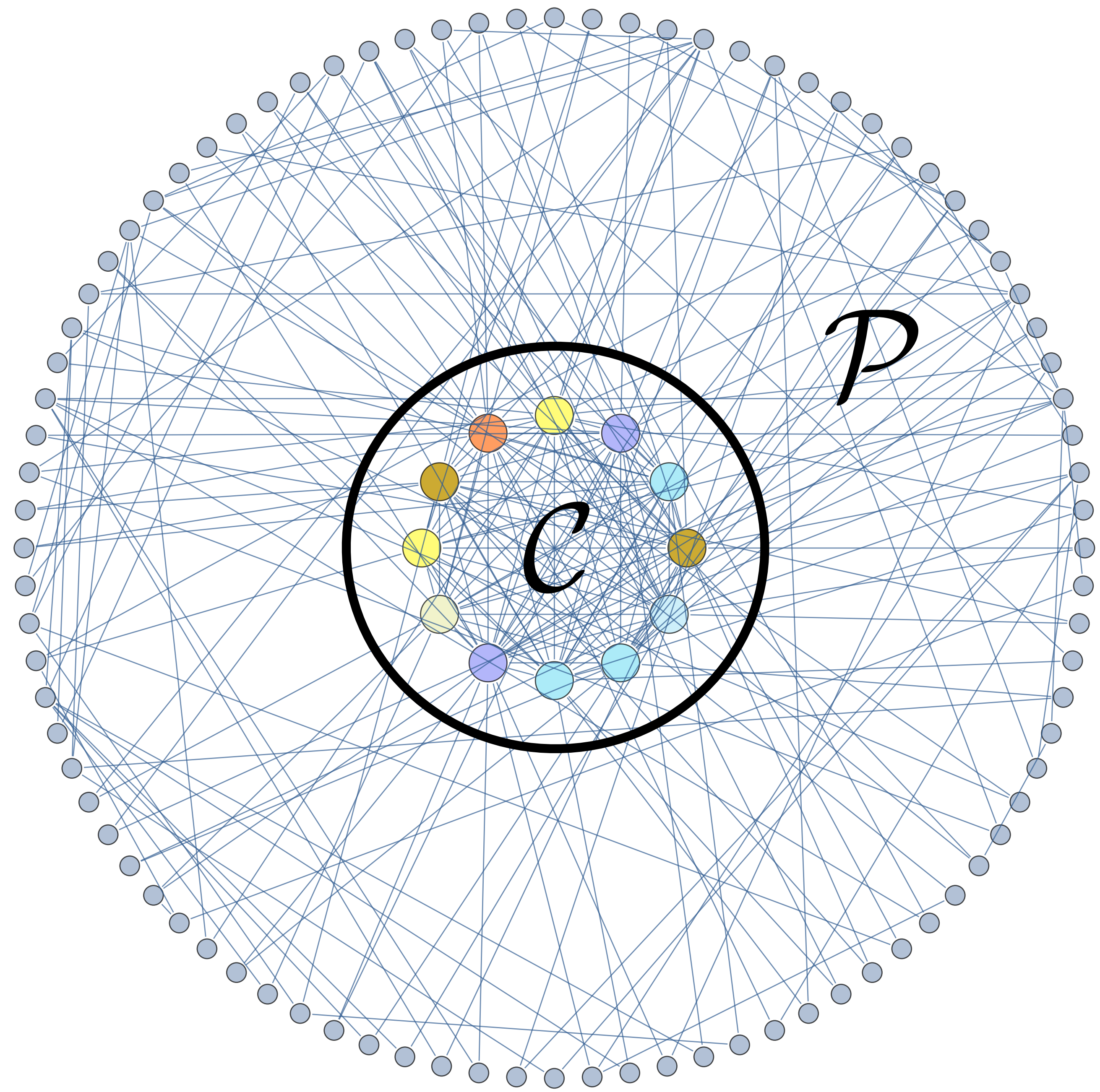
Core:
small, dense



Core-Periphery Structure

Core:
small, dense

Periphery:
large, sparse



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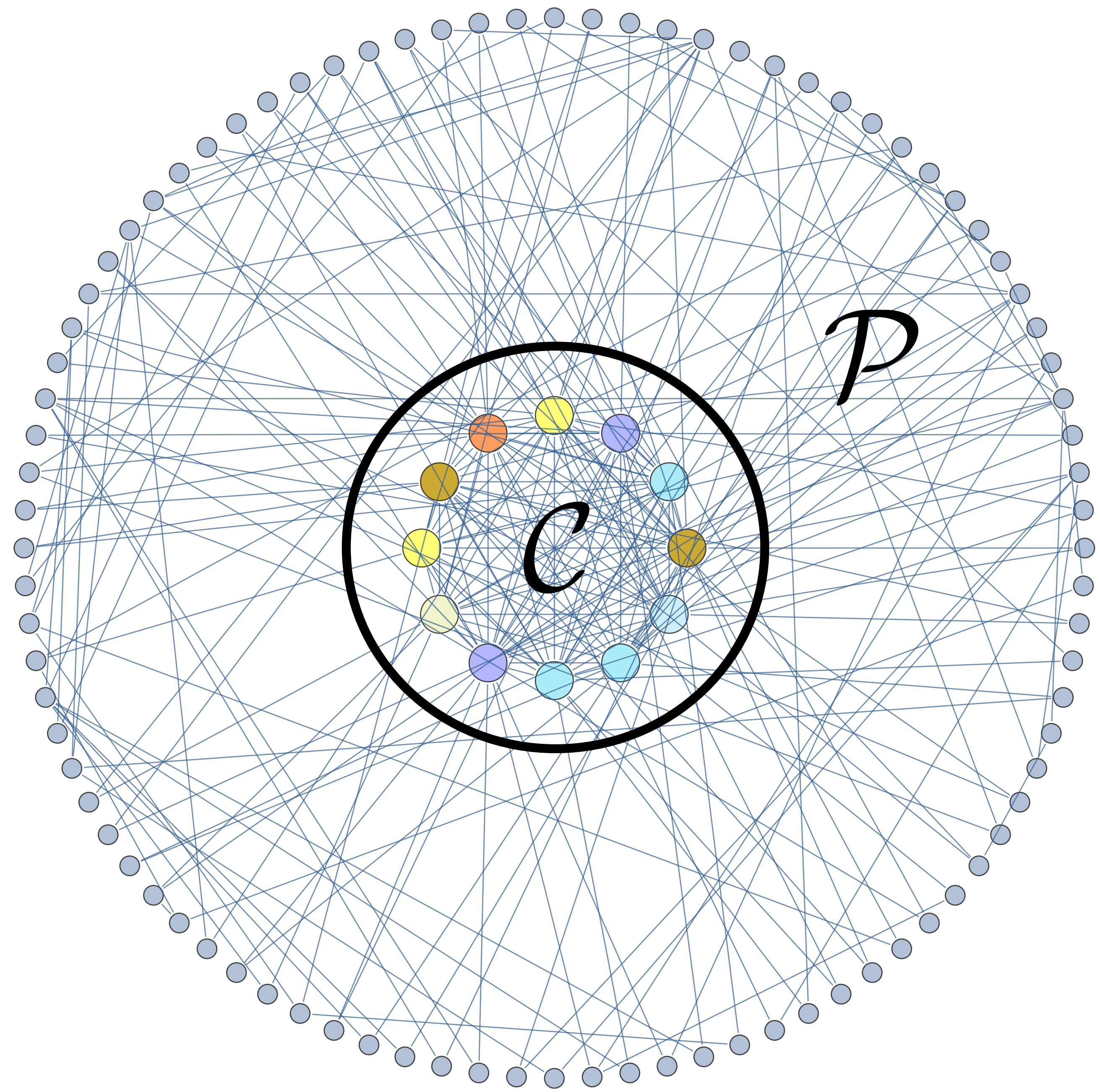
- Social networks structure
  [Avin, Lotker, Pignolet,Turkel 2012]



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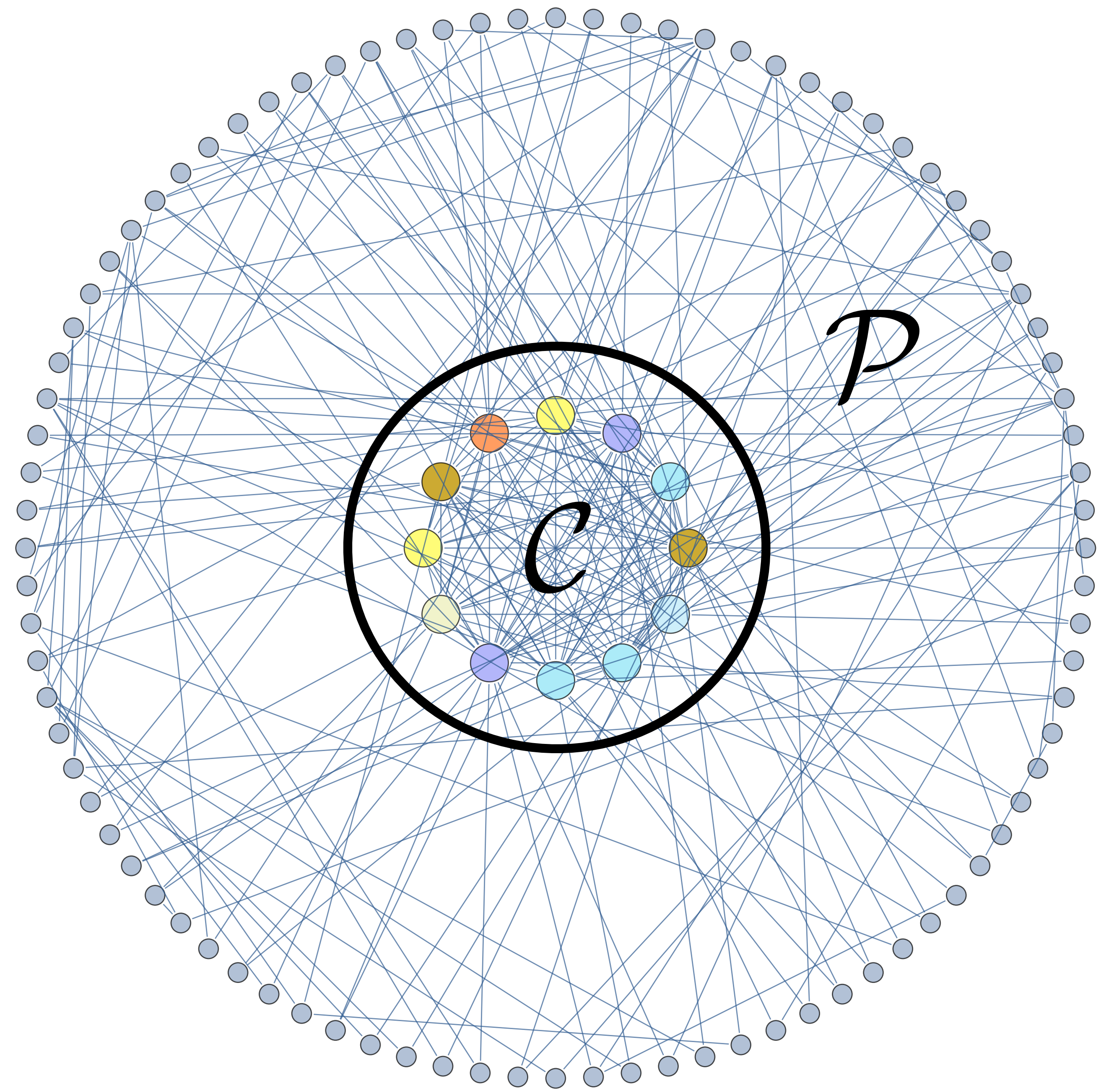


- Social networks structure
  [Avin, Lotker, Pignolet,Turkel 2012]

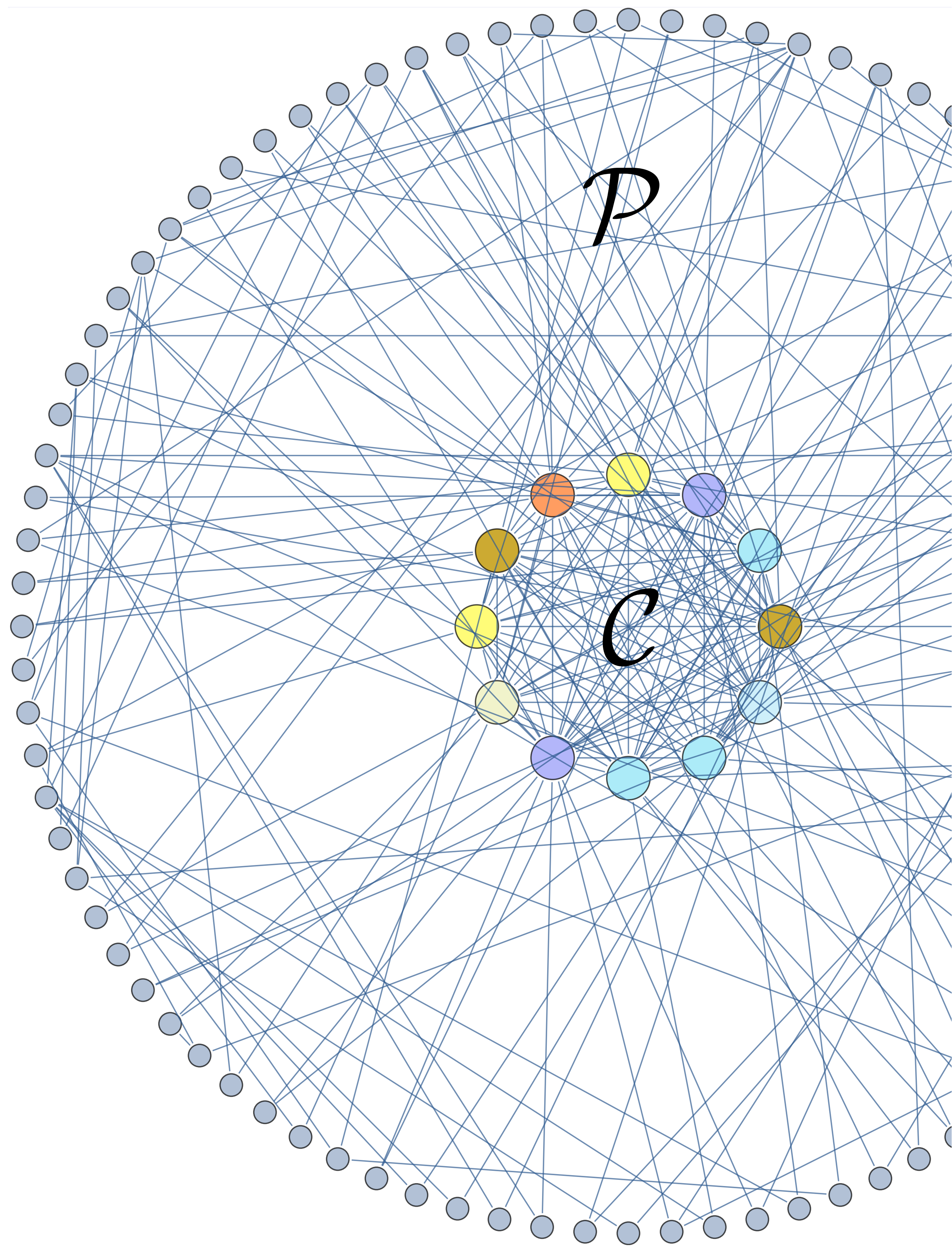
- Global economy - wealthiest countries are well connected with trade and transportation routes

- P2P networks - (e.g., Skype where super nodes are the Core)

Axiomatic Approach

- We define networks using axioms:
  - No concrete generative model
  - Abstract away algorithmic requirements
  - Algorithmic vs structural properties
  - Models that satisfy axioms can be proposed

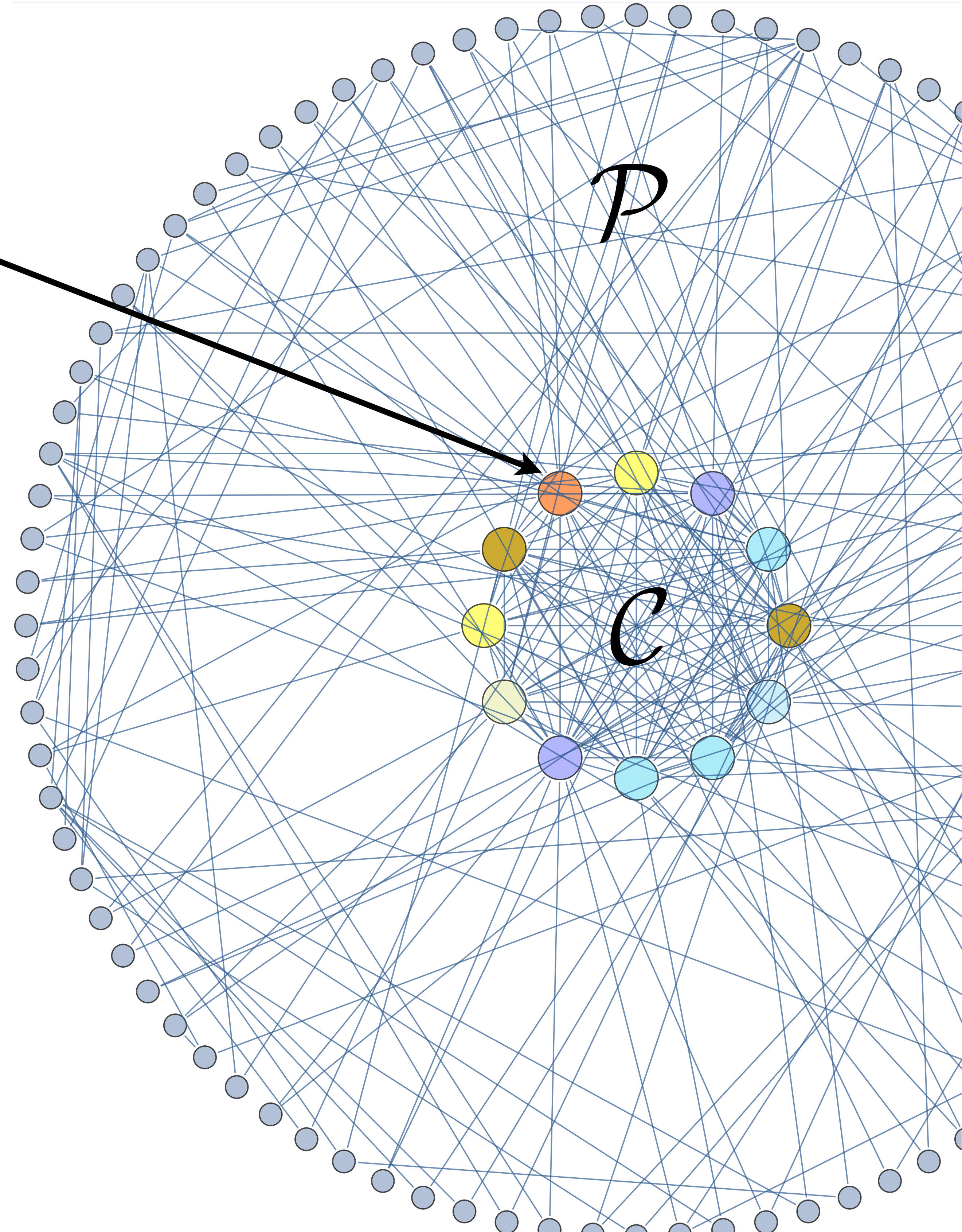
Core-Periphery Axioms: A1, A2, A3



Core-Periphery Axioms: A1, A2, A3

A1: **Balanced *Core* Boundary**

$$\forall v \in \mathcal{C}, \quad \frac{d_{\text{out}}(v)}{d_{\text{in}}(v)} = O(1)$$



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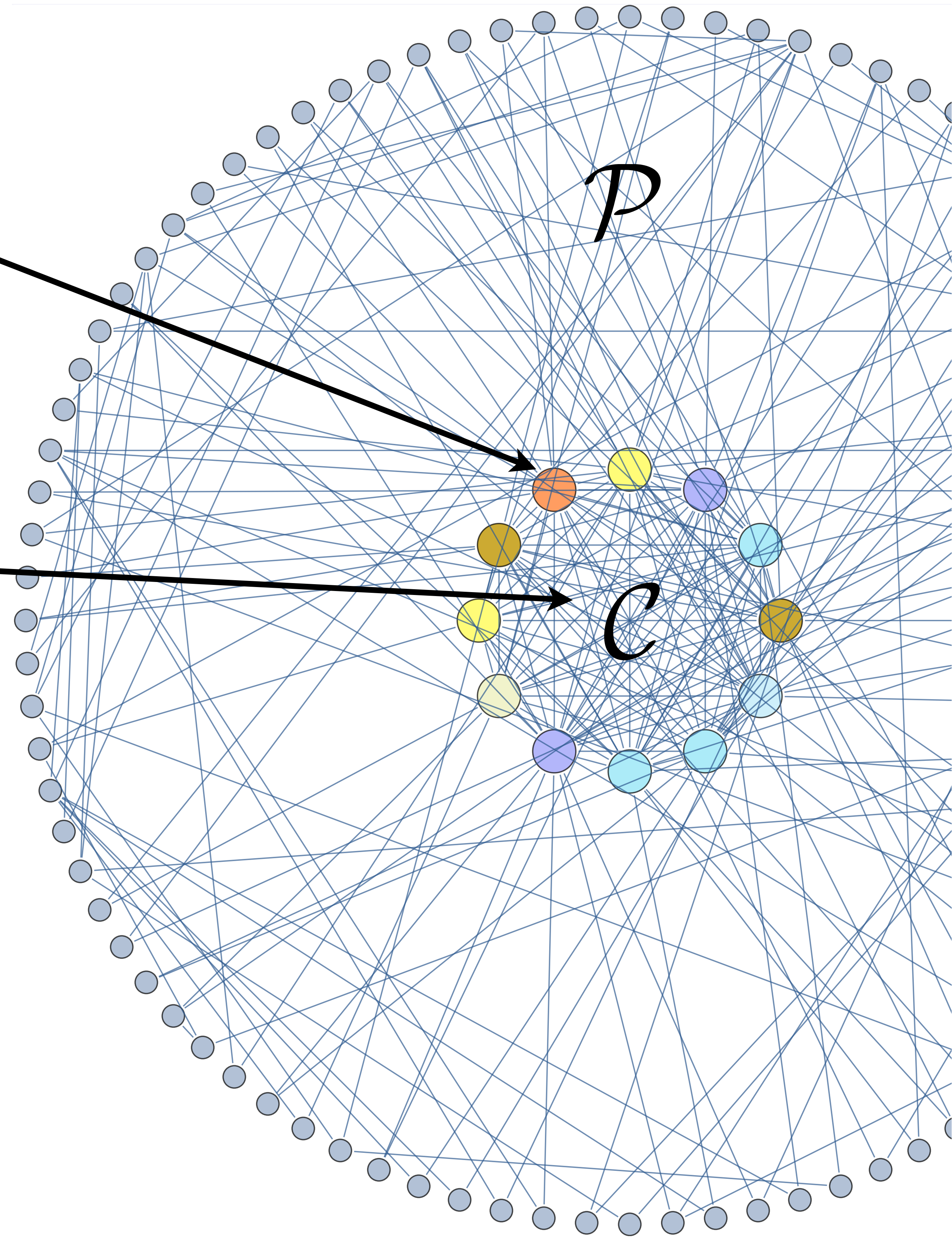
A1: **Balanced *Core* Boundary**

$$\forall v \in \mathcal{C}, \quad \frac{d_{\text{out}}(v)}{d_{\text{in}}(v)} = O(1)$$

A2: ***Core* Clique Emulation**

$$\mathcal{C} \xleftrightarrow{\text{all-to-all}} \mathcal{C}, \text{ in } O(1)$$



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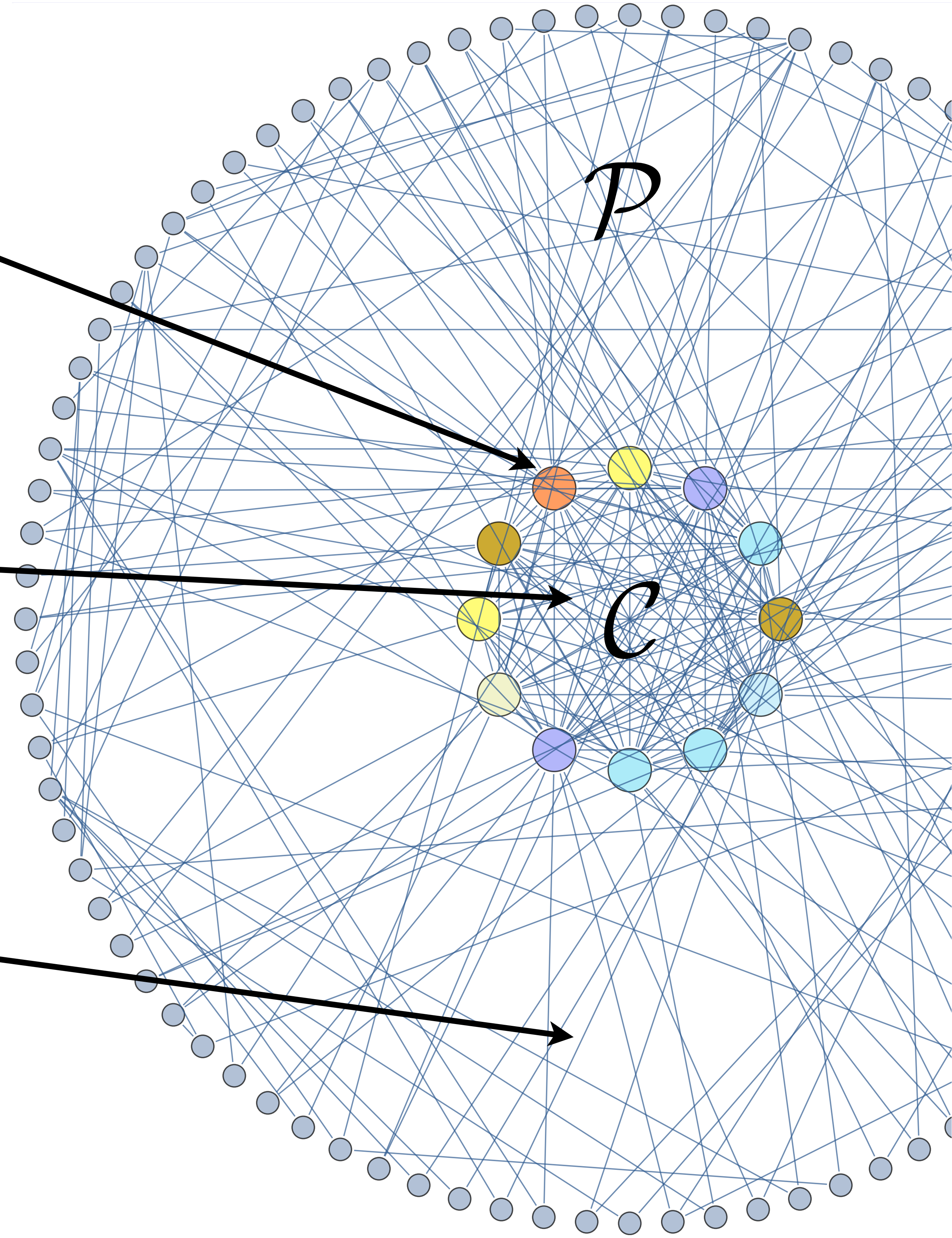
A3: ***Periphery-Core* Convergecast**

$$\mathcal{P} \xrightarrow{\text{all-to-any}} \mathcal{C}, \ \text{in } O(1)$$



Axiom Independence

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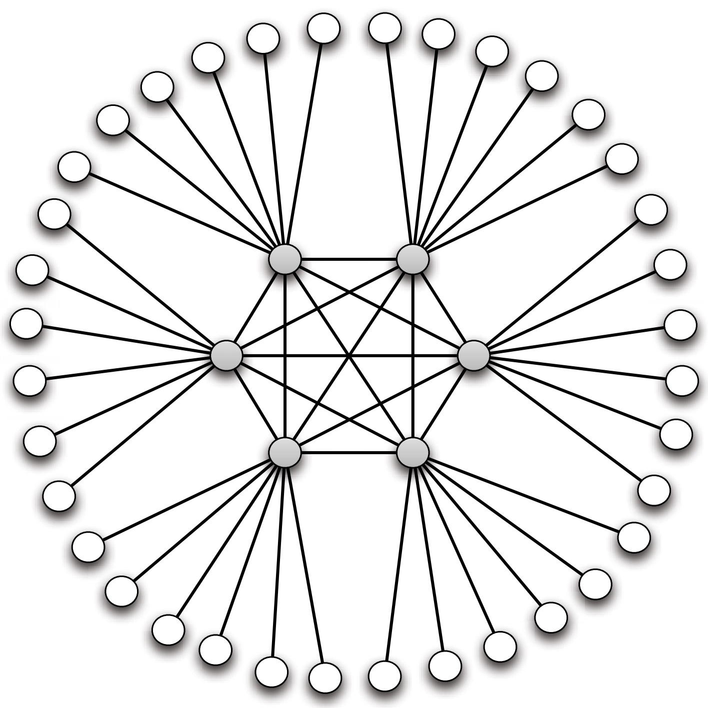

A1 A2 A3

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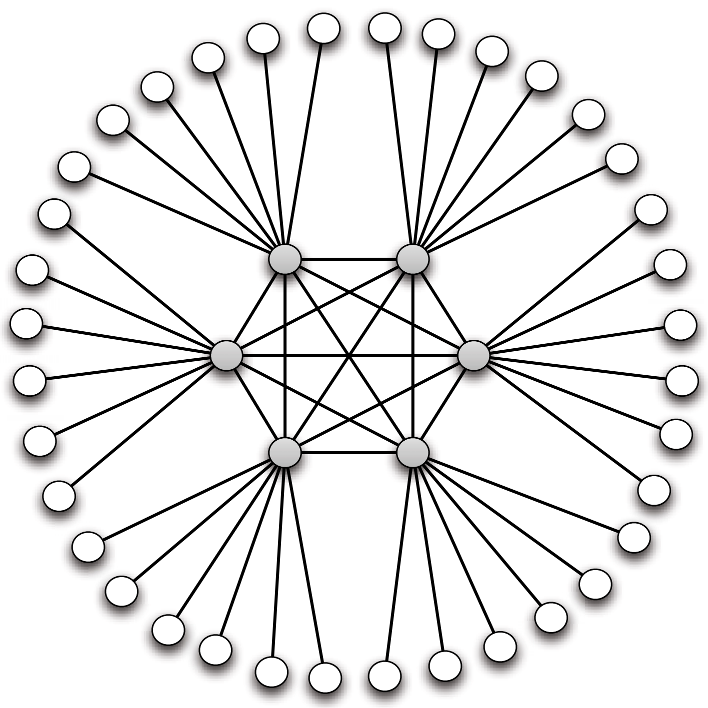

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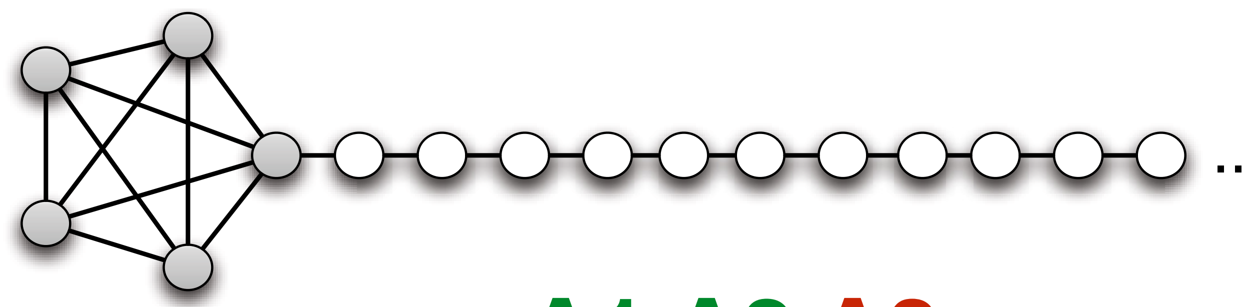

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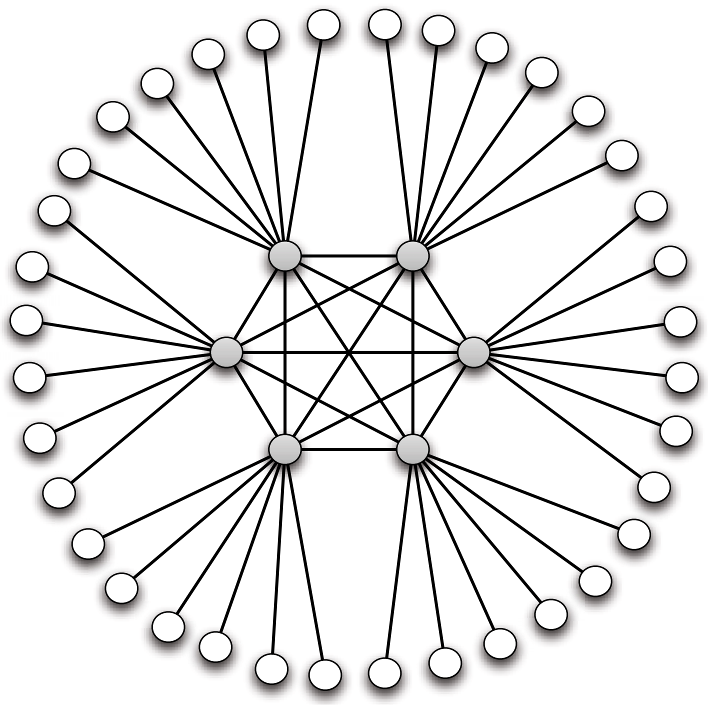
Axiom Independence



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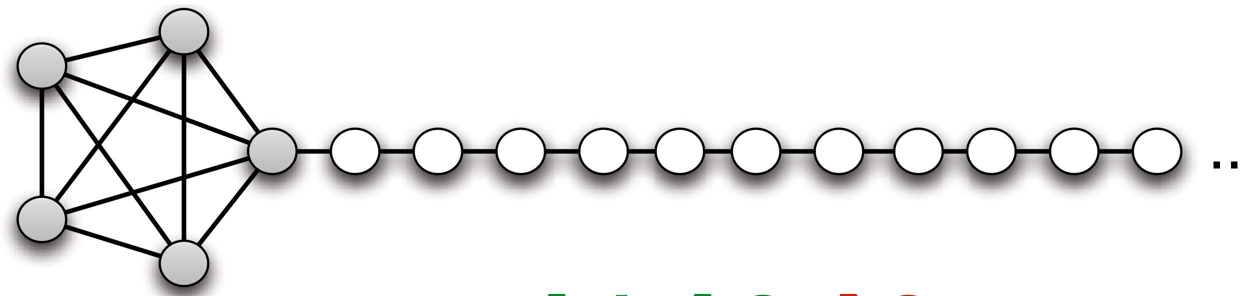

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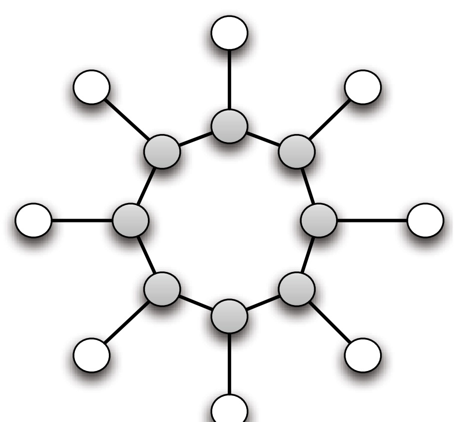
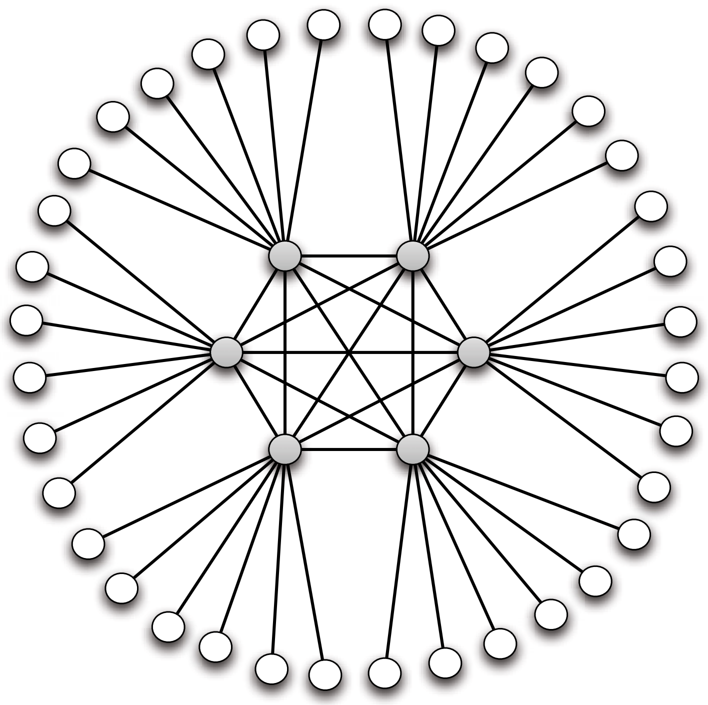


A1 A2 A3

no clique emulation

Axiom Independence



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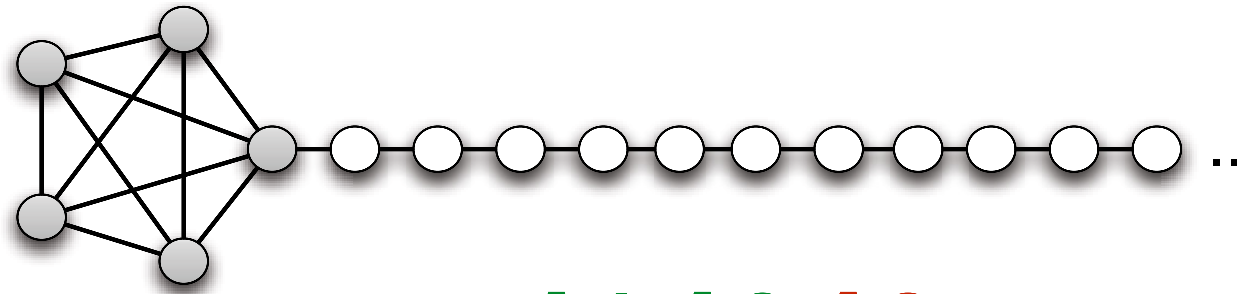

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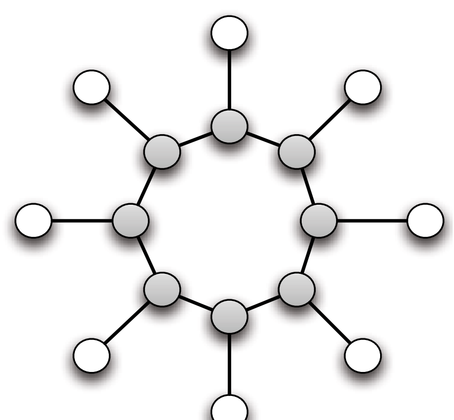

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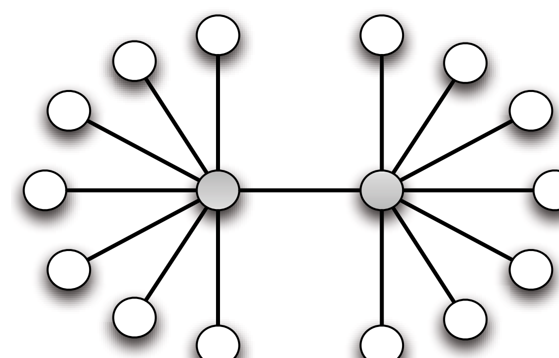
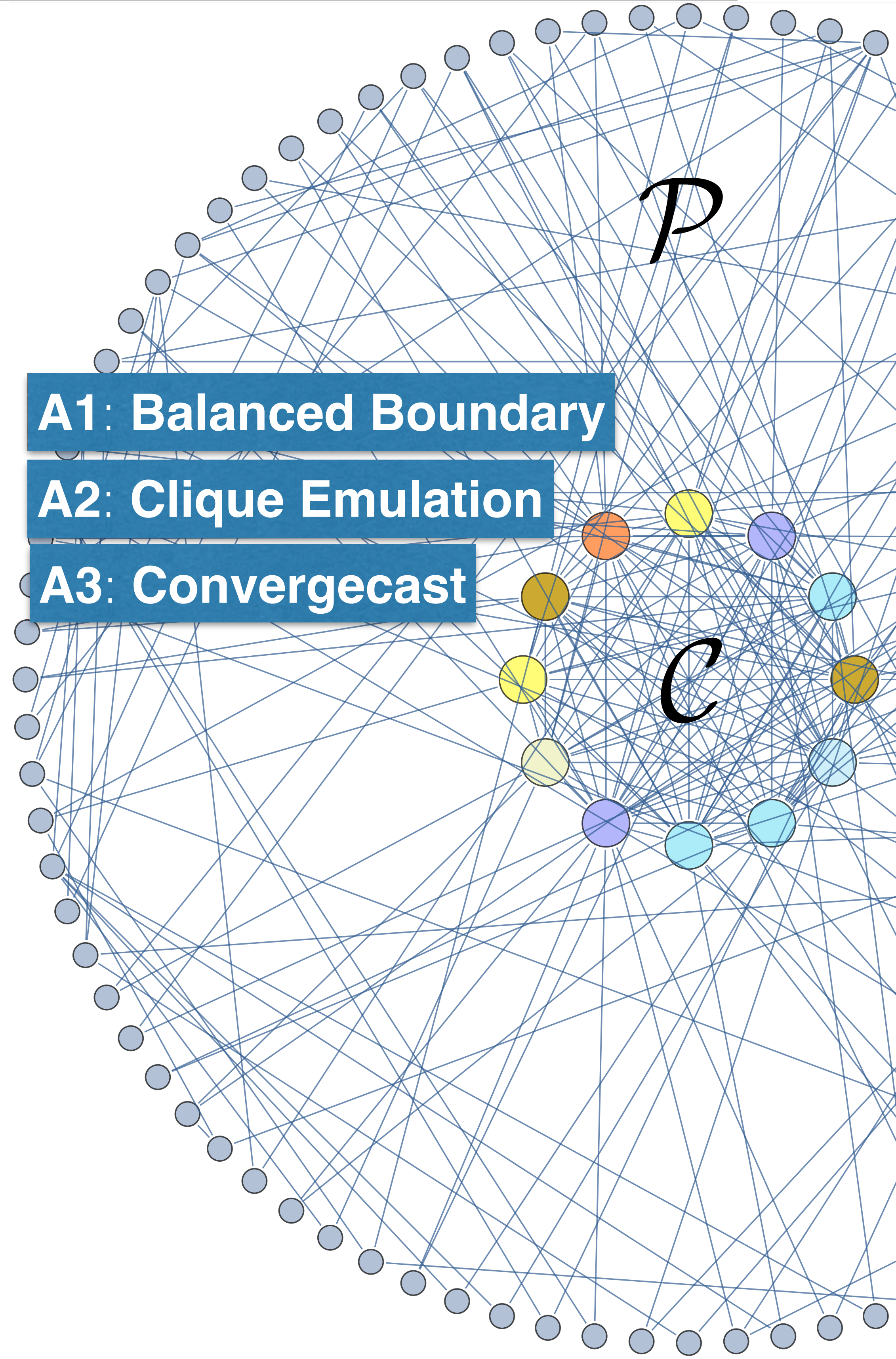


A1 A2 A3
no balanced boundary

Structural Implications

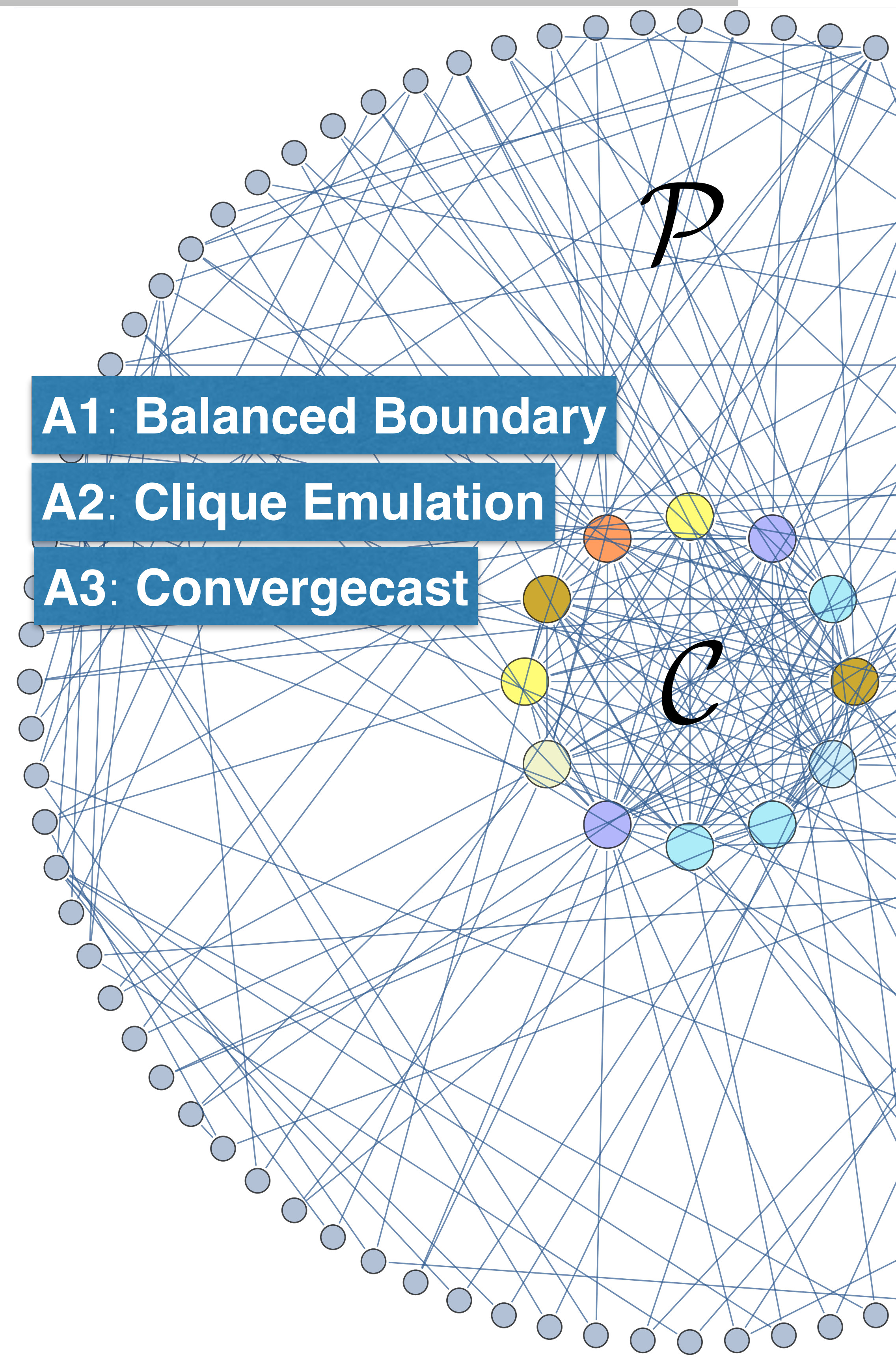


Structural Implications

Balanced boundary

$$\forall v \in \mathcal{C}, d_{\text{out}}(v) = O(n_{\mathcal{C}}) \quad \textbf{A1}$$

$$d_{\text{in}}(v) = \Theta(n_{\mathcal{C}}) \quad \textbf{A2}$$



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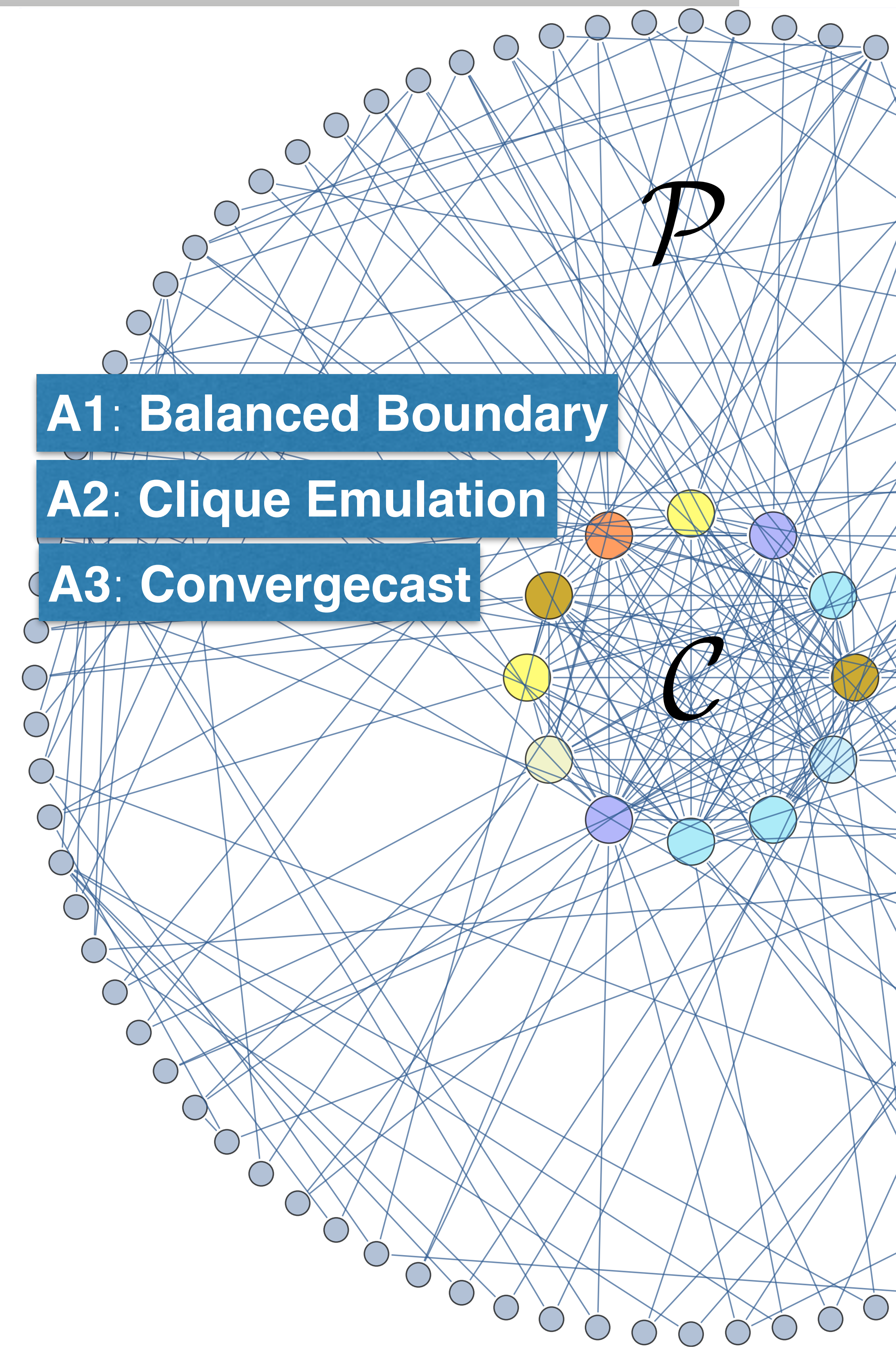
***Core* is dense** A2

$$m_{\mathcal{C}} = \Theta(n_{\mathcal{C}}^2)$$



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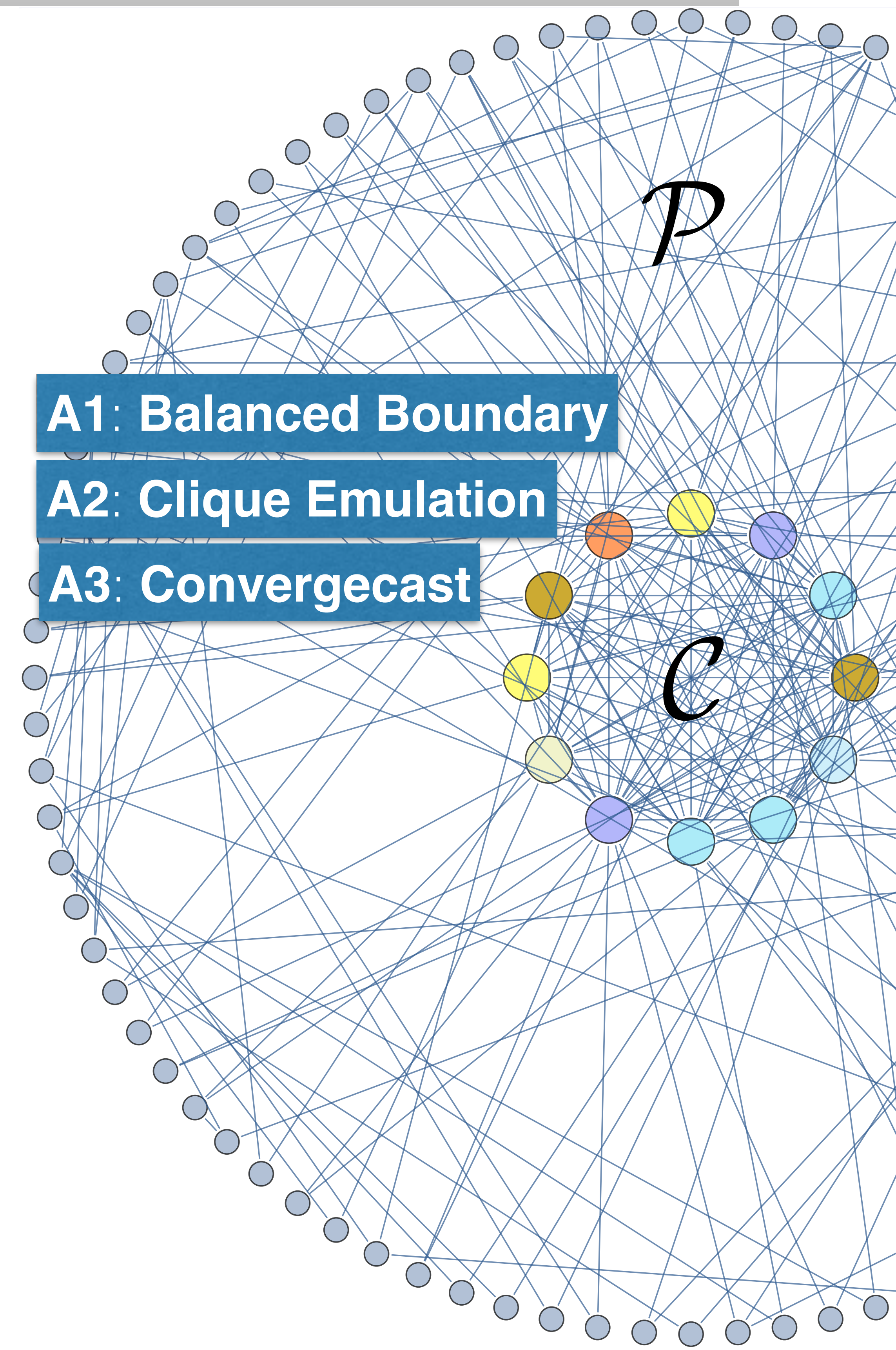
Constant Diameter **A2,A3**



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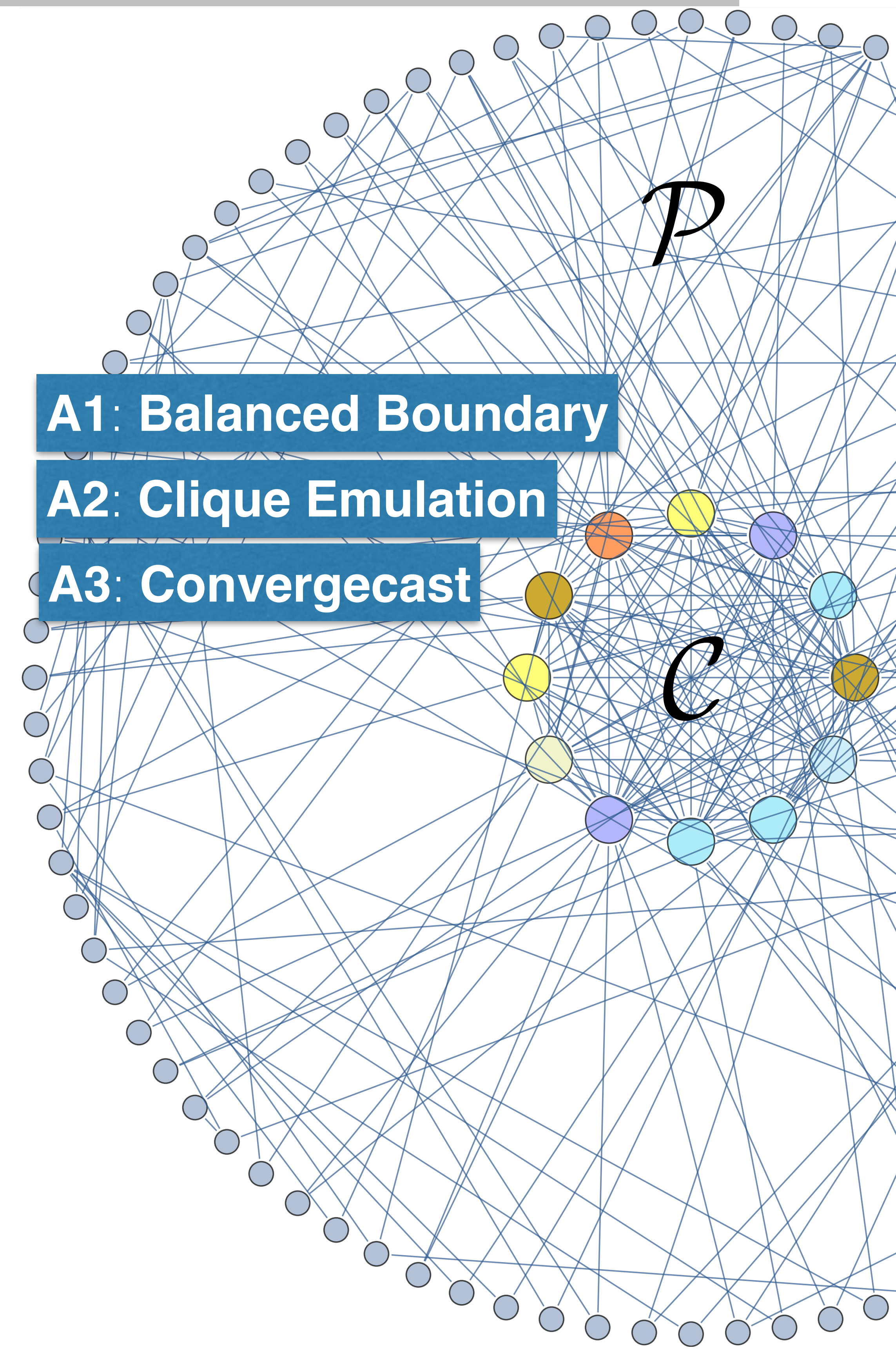
Constant Diameter A2,A3

Size of the *Core*

$$\Omega(\sqrt{n}) \leq n_{\mathcal{C}} \leq O(\sqrt{m})$$



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Constant Diameter

A2,A3

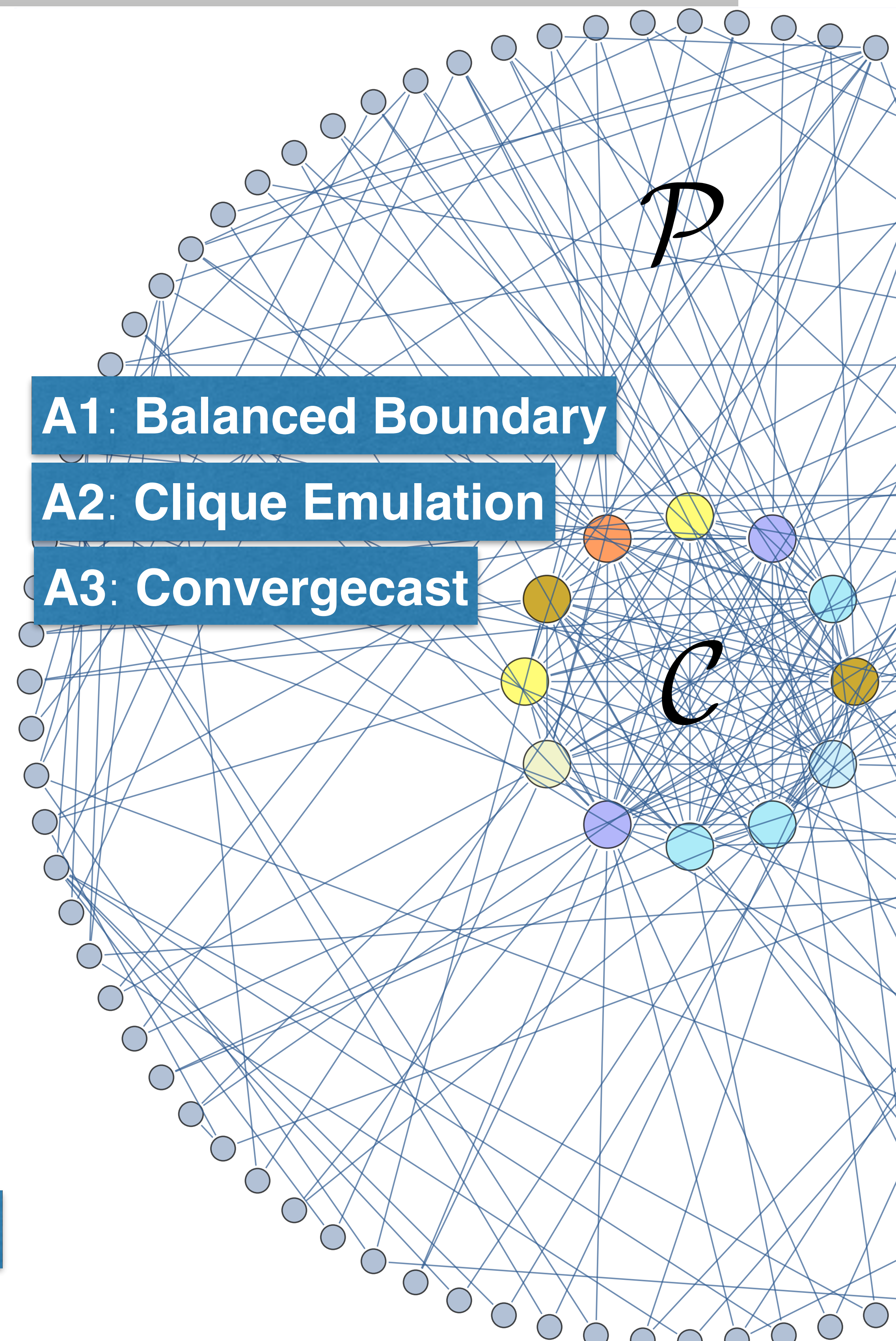
Size of the *Core*

$$\Omega(\sqrt{n}) \leq n_{\mathcal{C}} \leq O(\sqrt{m})$$

$$\sum_{v \in \mathcal{C}} d_{\text{in}}(v) \leq 2m$$

$$n_{\mathcal{C}}^2 \leq 2m$$ A2

$$n_{\mathcal{C}} \leq O(\sqrt{m})$$



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Constant Diameter

A2,A3

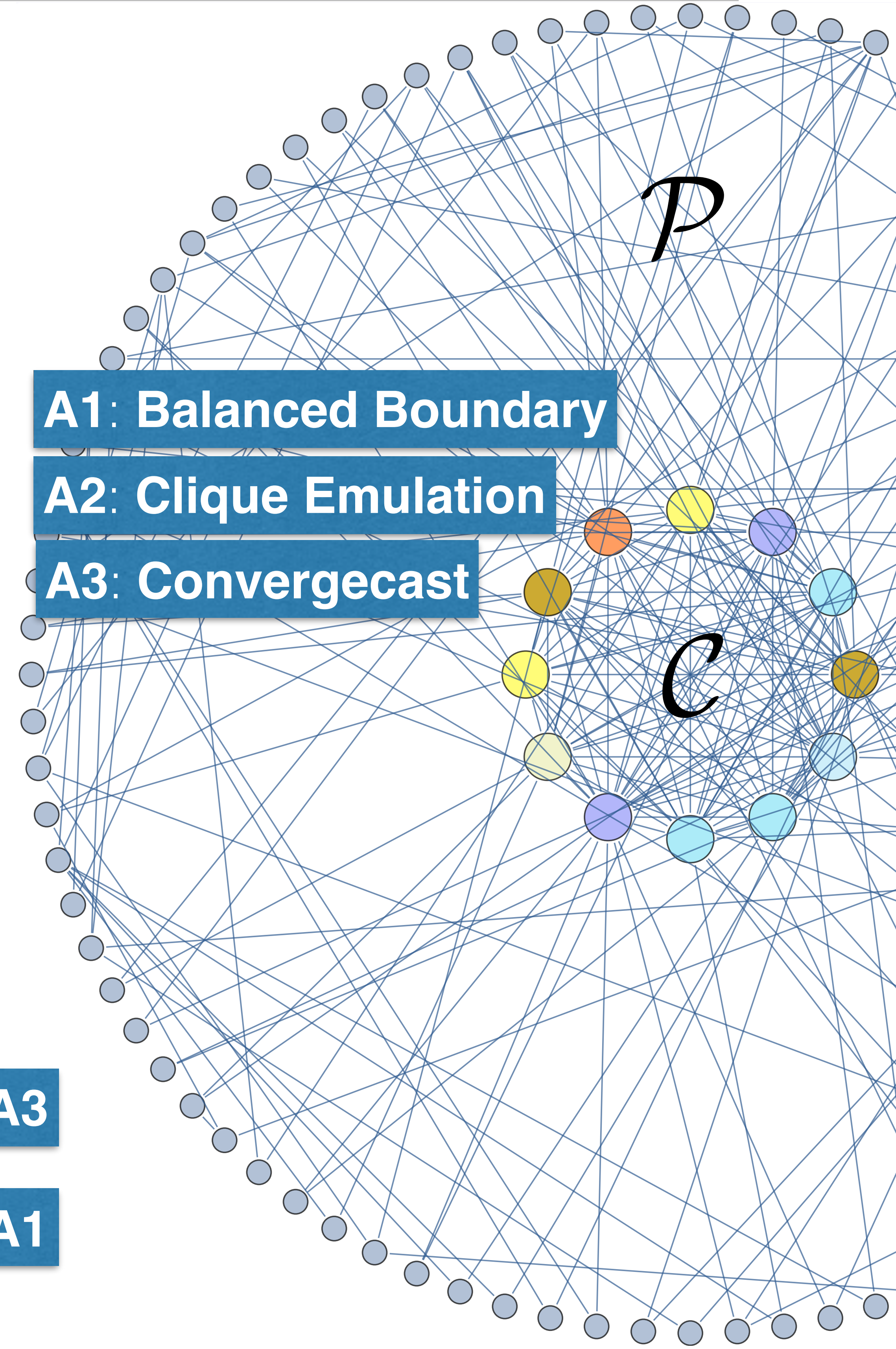
Size of the *Core*

$$\Omega(\sqrt{n}) \leq n_{\mathcal{C}} \leq O(\sqrt{m})$$

$$\sum_{v \in \mathcal{C}} d_{\text{out}}(v) \geq n - n_{\mathcal{C}}$$ A3

$$n_{\mathcal{C}}^2 \geq n - n_{\mathcal{C}}$$ A1

$$n_{\mathcal{C}} \geq \Omega(\sqrt{n})$$



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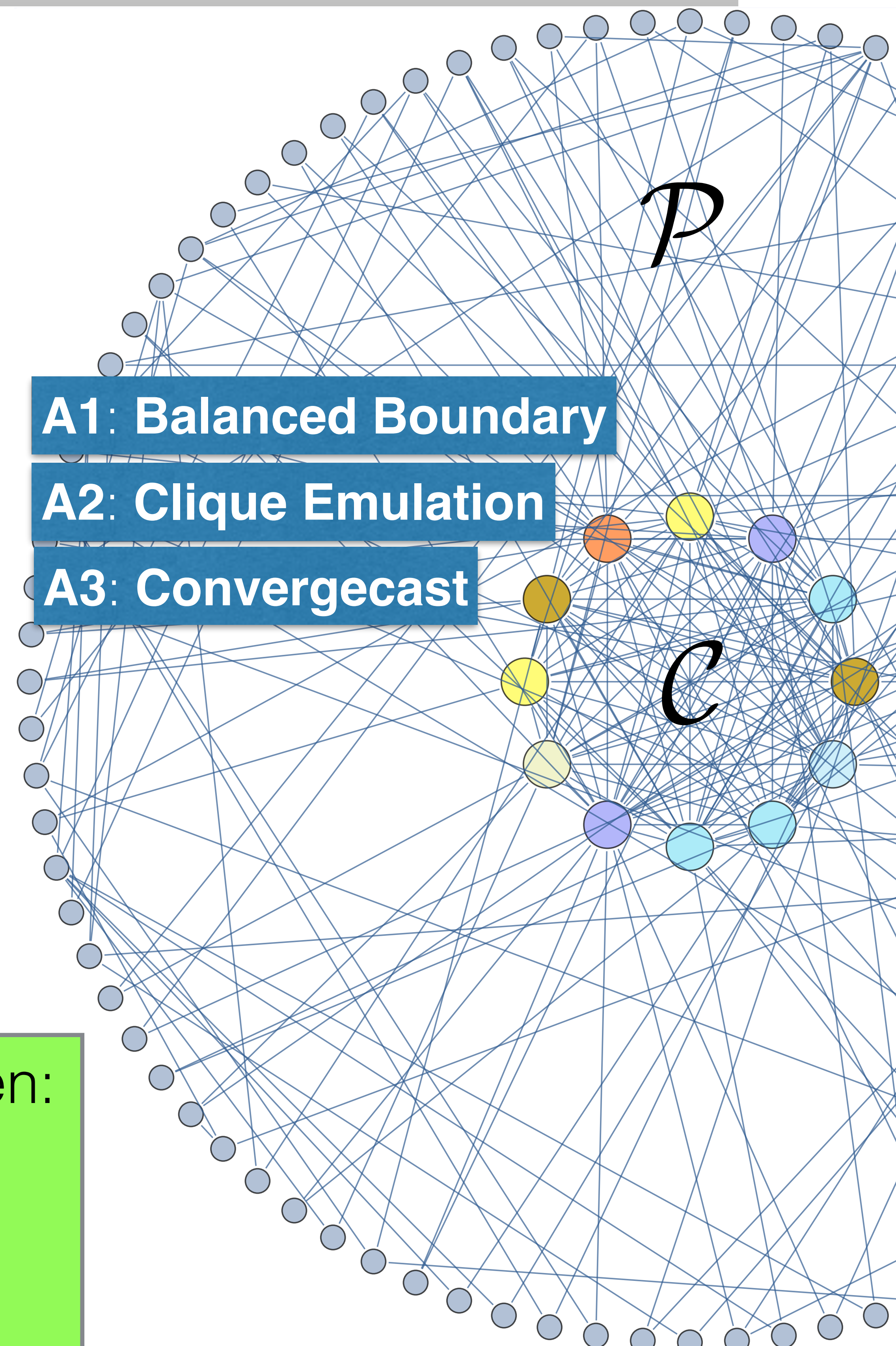
Constant Diameter **A2,A3**

Size of the *Core*

$\Omega(\sqrt{n}) \leq n_{\mathcal{C}} \leq O(\sqrt{m})$

if *m* is linear, then:

$$n_{\mathcal{C}} = \Theta(\sqrt{n})$$
$$m_{\mathcal{C}} = \Theta(m)$$



Algorithms for Core-Periphery Networks

Matrix Operations

$A, B \in \mathbb{Z}^{n \times n}$, $O(k)$ sparse

$s \in \mathbb{Z}^n$

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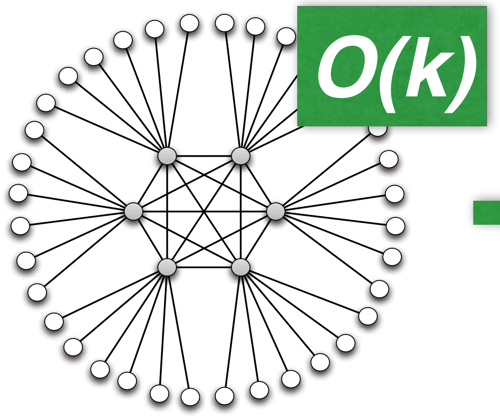
A →  → A^T

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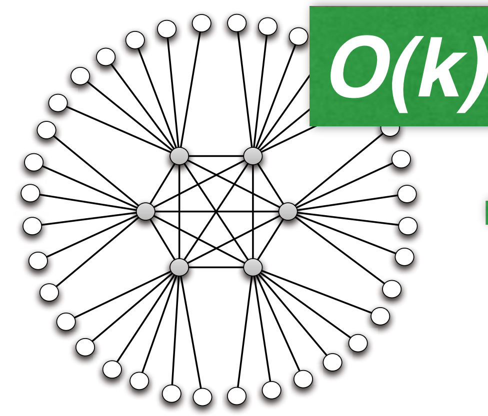
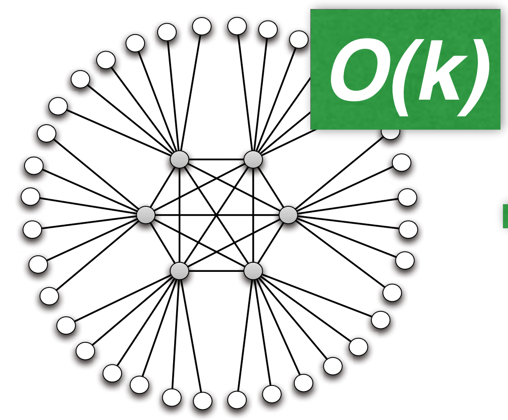


A^T

s, A



$s' = sA$

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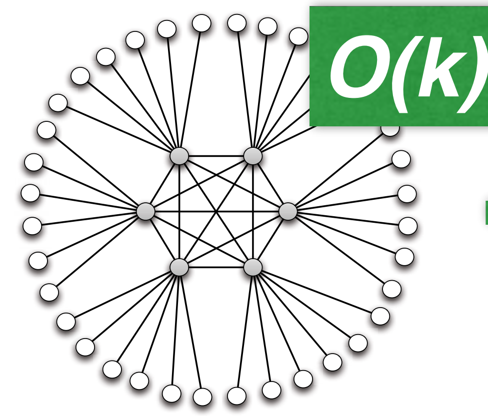
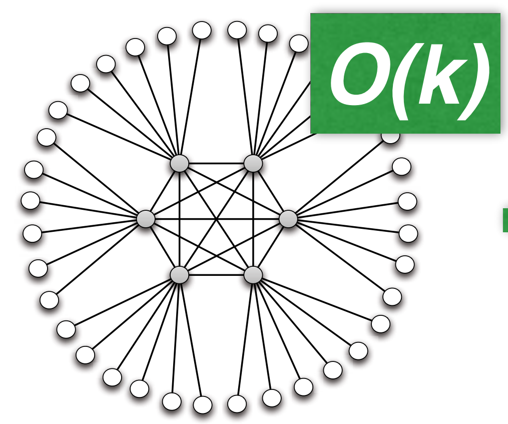
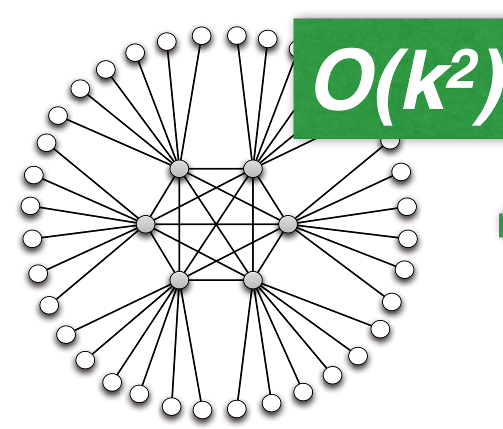
A →  → A^T

s, A →  → $s' = sA$

A, B →  → $C = AB$

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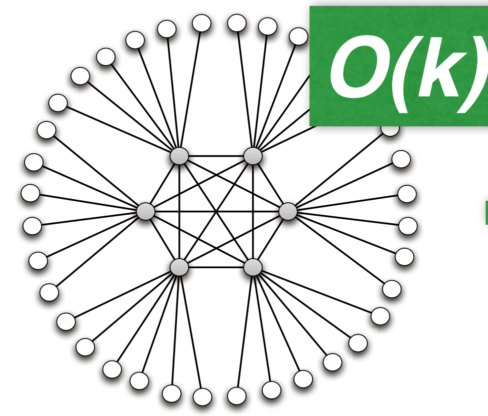
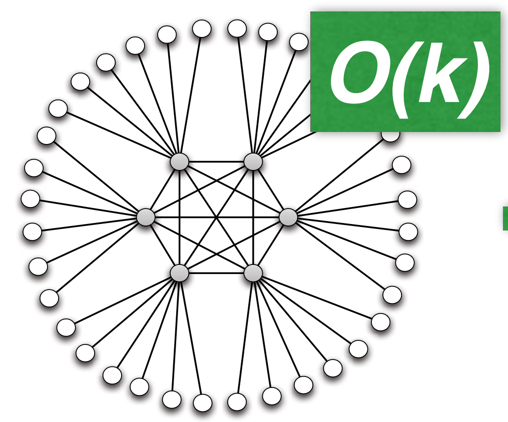
Memory per node - $O(\sqrt{n})$

A →



→ A^T

s, A →



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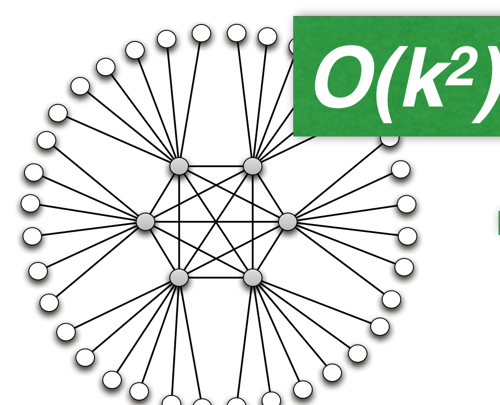
A, B →



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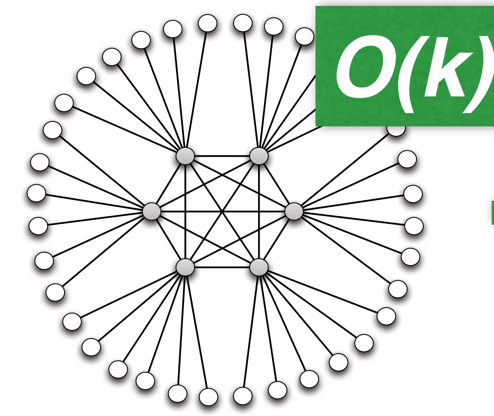
Based on fast info spreading in clique

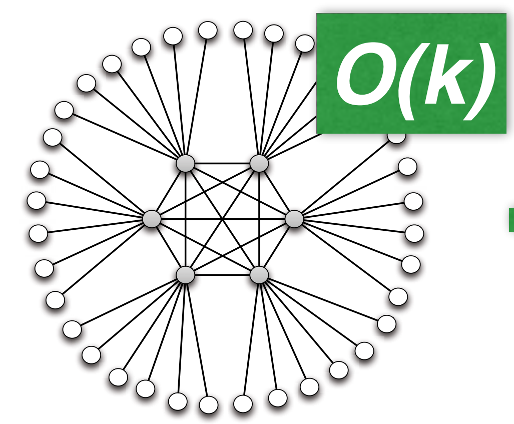
[Lenzen, Wattenhofer, 2011]

$A \rightarrow$



$\rightarrow A^T$

$s, A \rightarrow$



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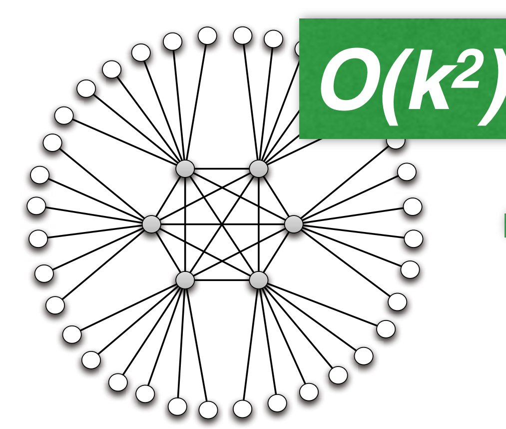


$\rightarrow C = AB$

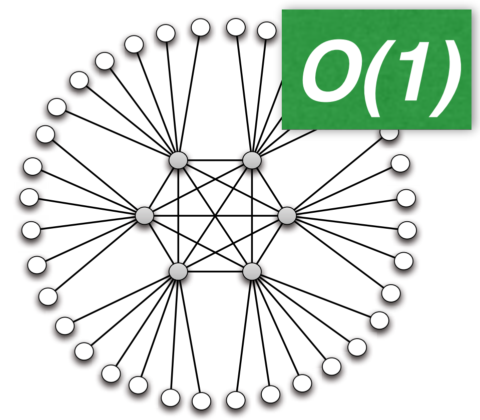
Algorithms for Core-Periphery Networks

Aggregate Functions

Algorithms for Core-Periphery Networks

Aggregate Functions

value for each node



each node knows:

rank of its value

mode of the values

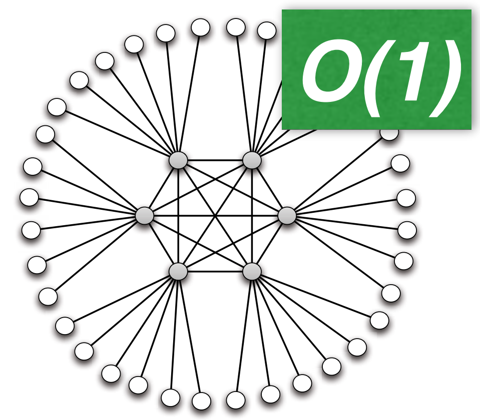
num of distinct values

median

Algorithms for Core-Periphery Networks

Aggregate Functions

value for each node



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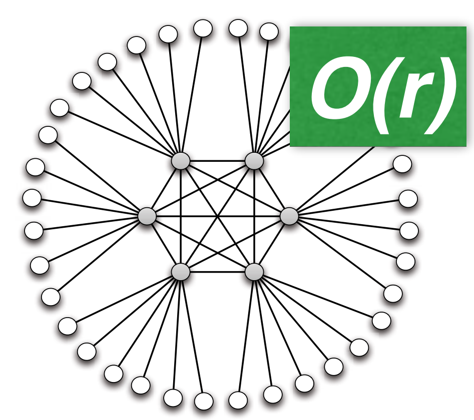
value for each node
each value belongs to
a specific group/area

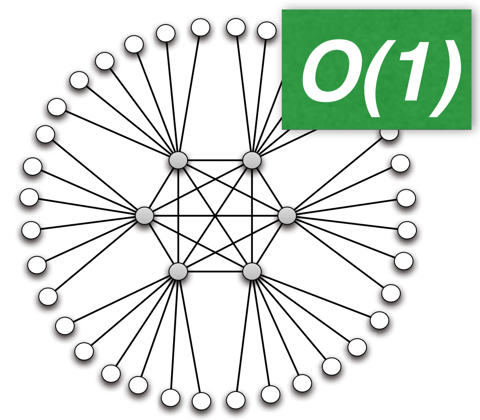


each node knows
top *r* of each area

Algorithms for Core-Periphery Networks

Aggregate Functions

value for each node



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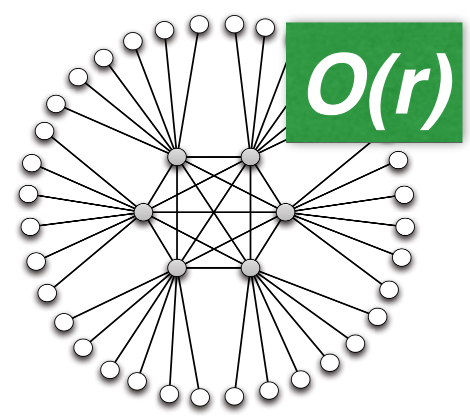

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Memory per node - $O(\sqrt{n})$

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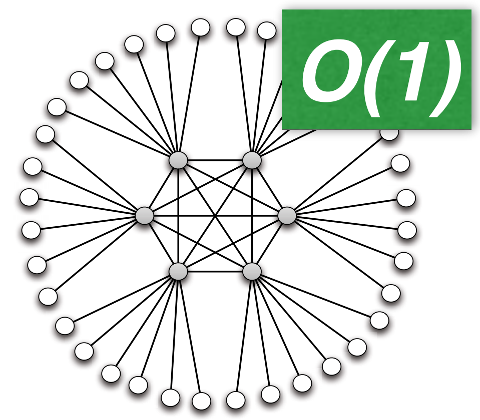

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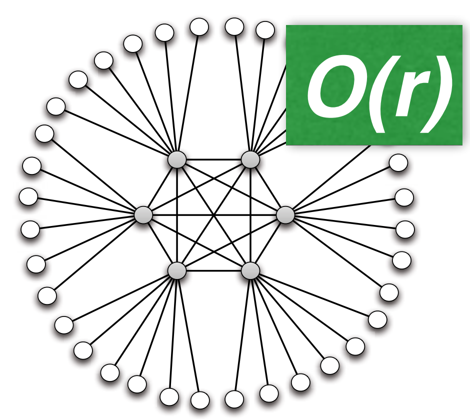

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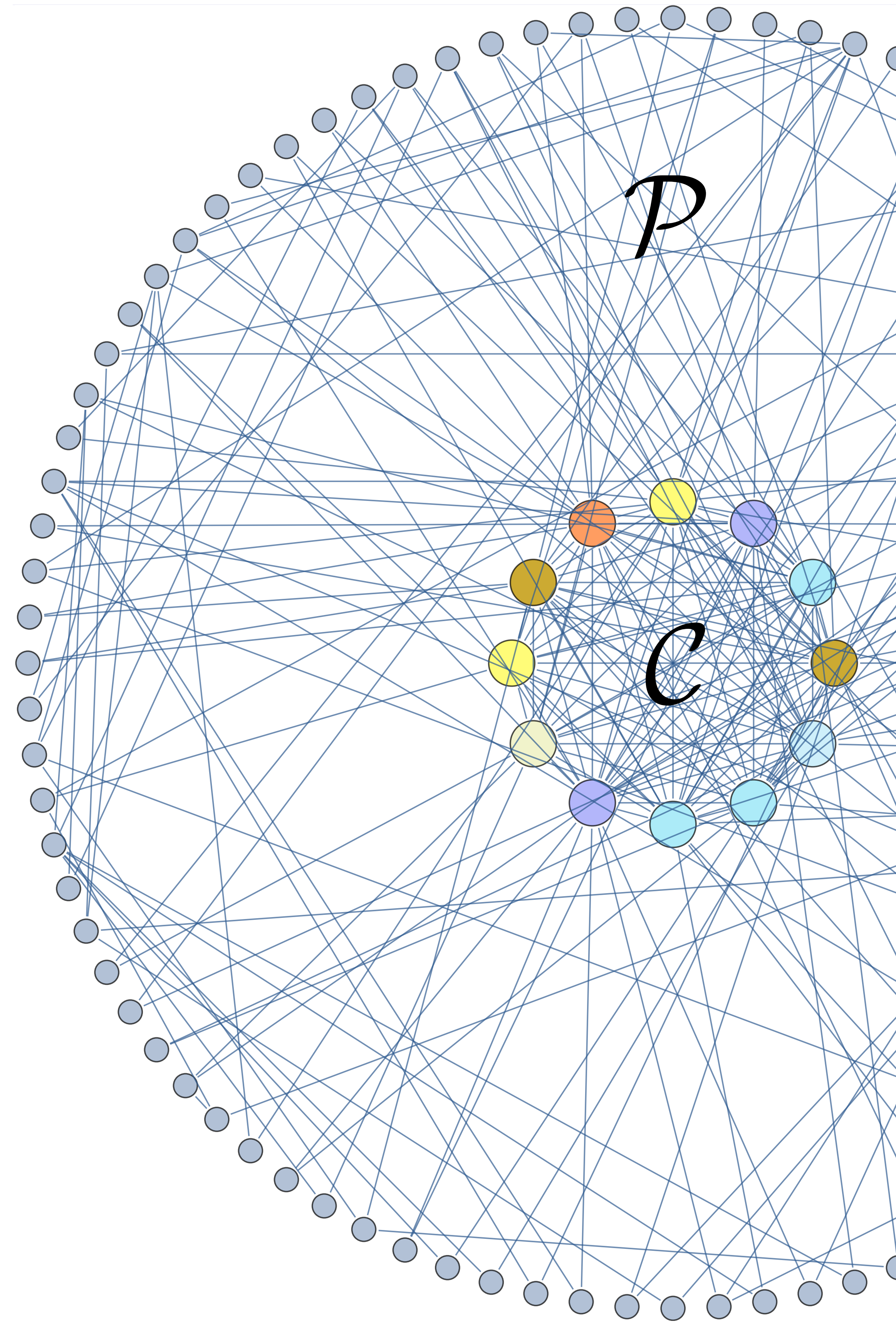
Based on fast sorting in clique
[Lenzen, 2013]

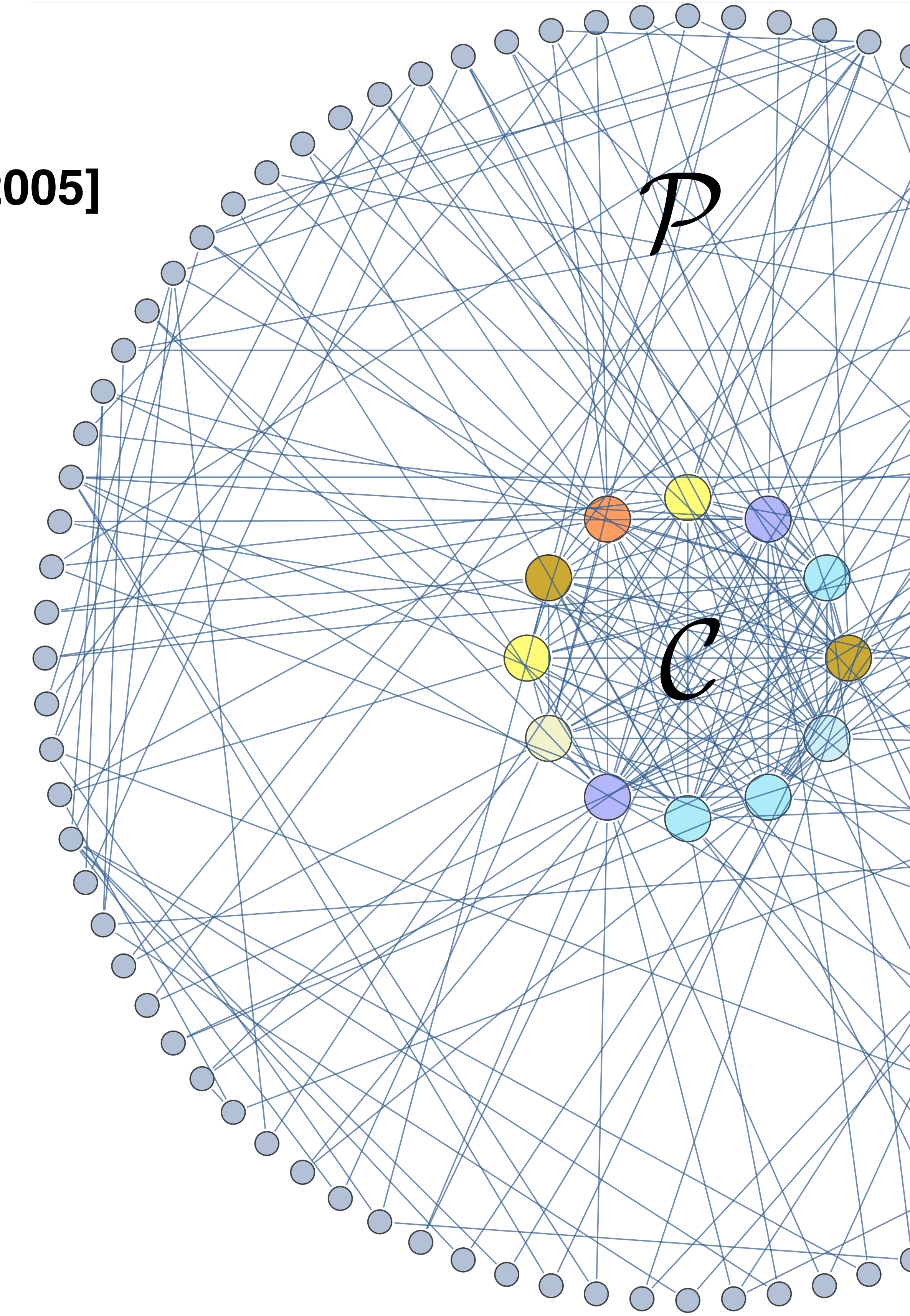
Distributed Minimum Spanning Tree (*CONGEST* Model)



Distributed Minimum Spanning Tree (*CONGEST* Model)

$D = 1 : \quad O(\log \log n)$

[Z. Lotker, B. Patt-Shamir, E. Pavlov, D. Peleg 2005]



Distributed Minimum Spanning Tree (*CONGEST* Model)

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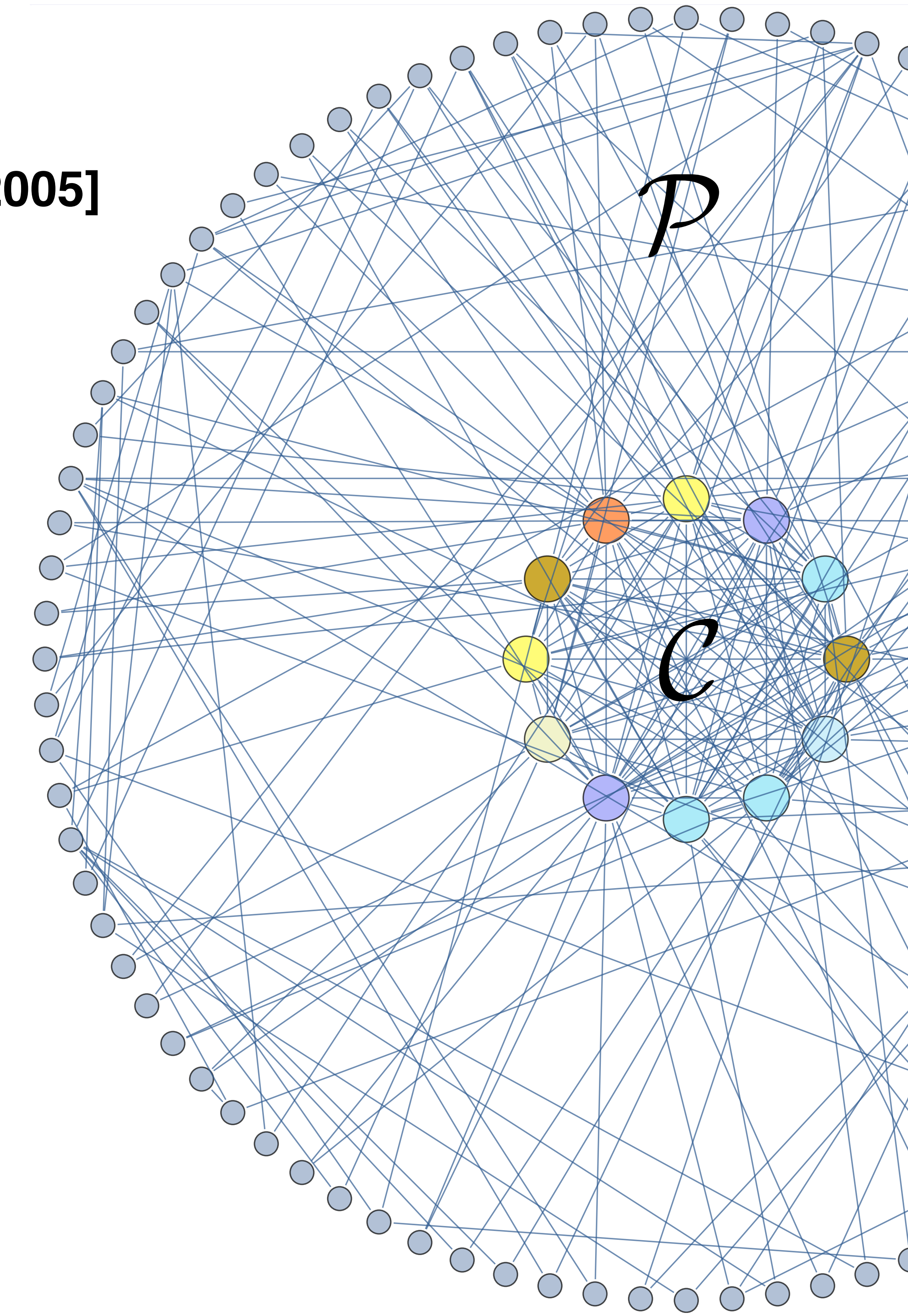

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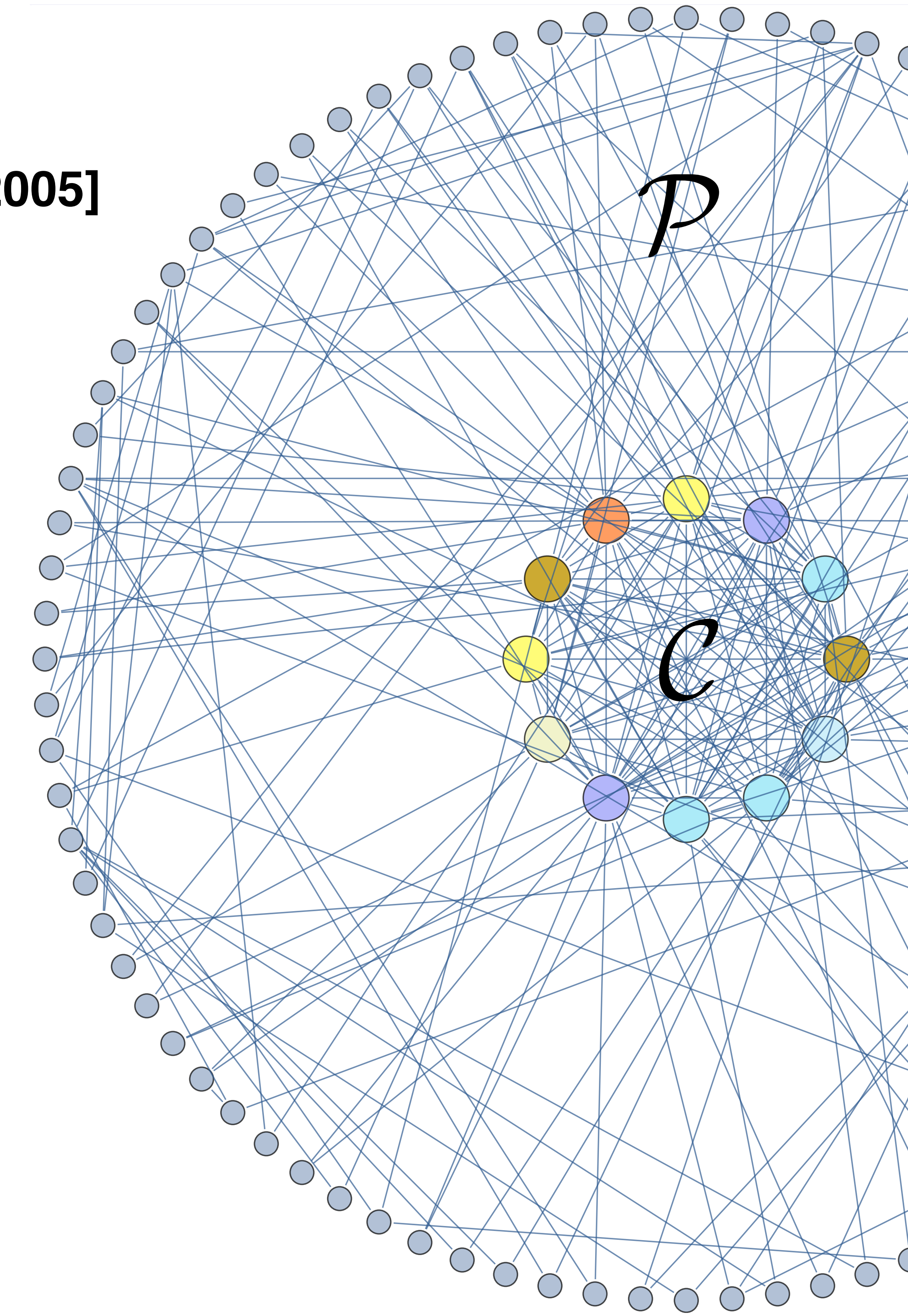
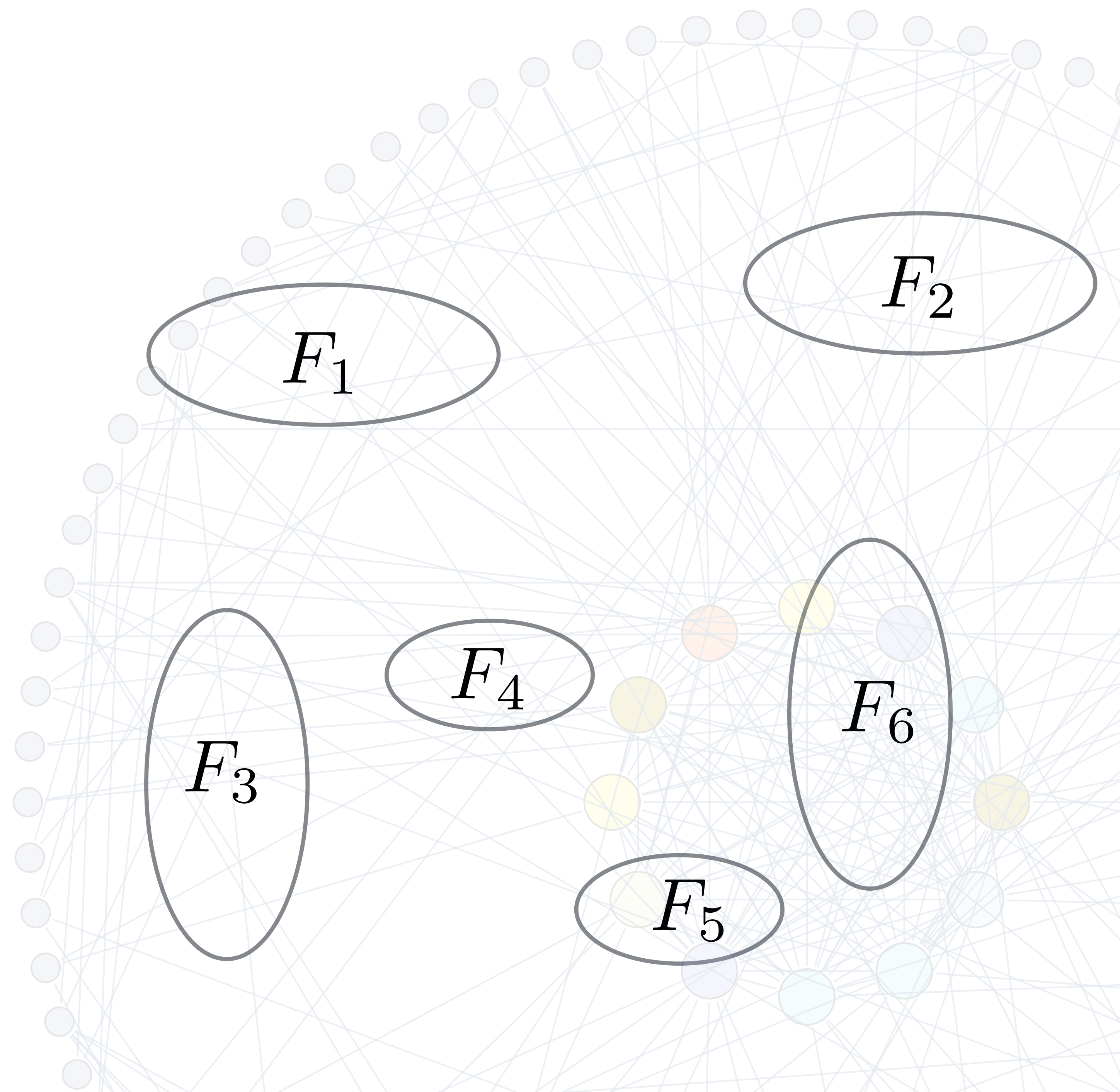
We prove:

$\mathcal{CP}$-MST algorithm: $\quad O(\log^2 n)$



Distributed MST for Core-Periphery Networks

- Based on Boruvka's algorithm - *log n* phases



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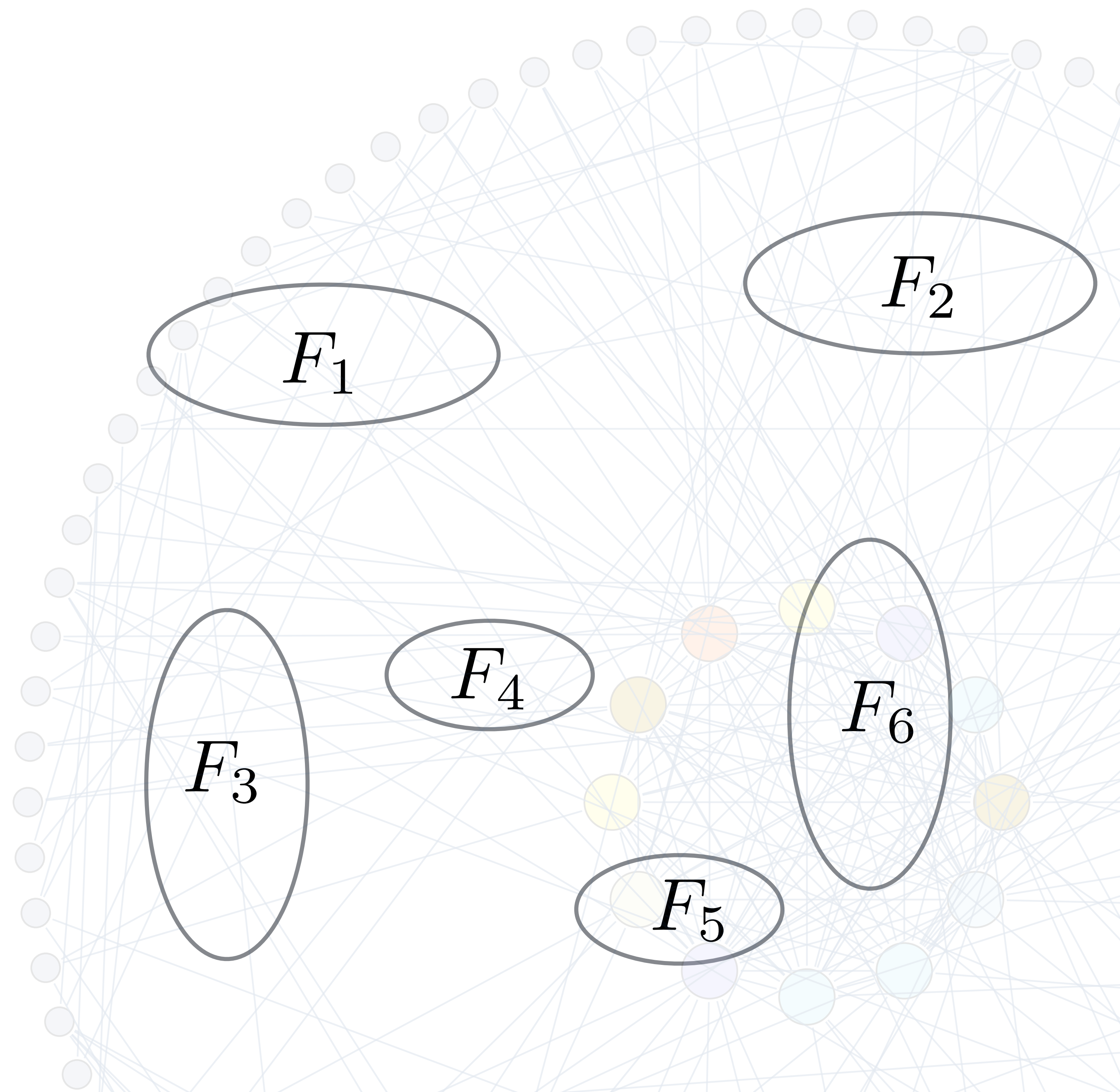
- On each phase need to exchange many messages:



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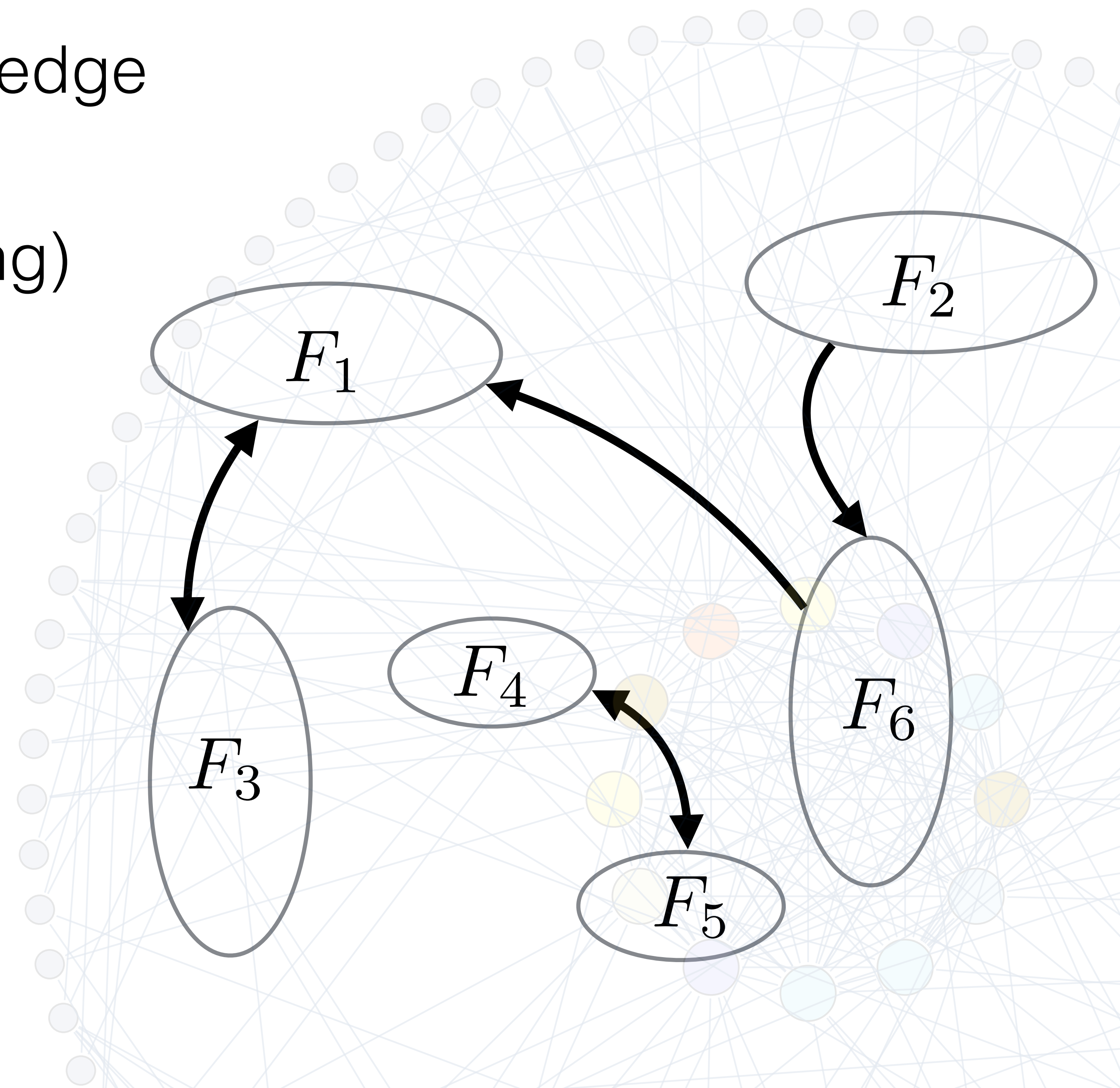

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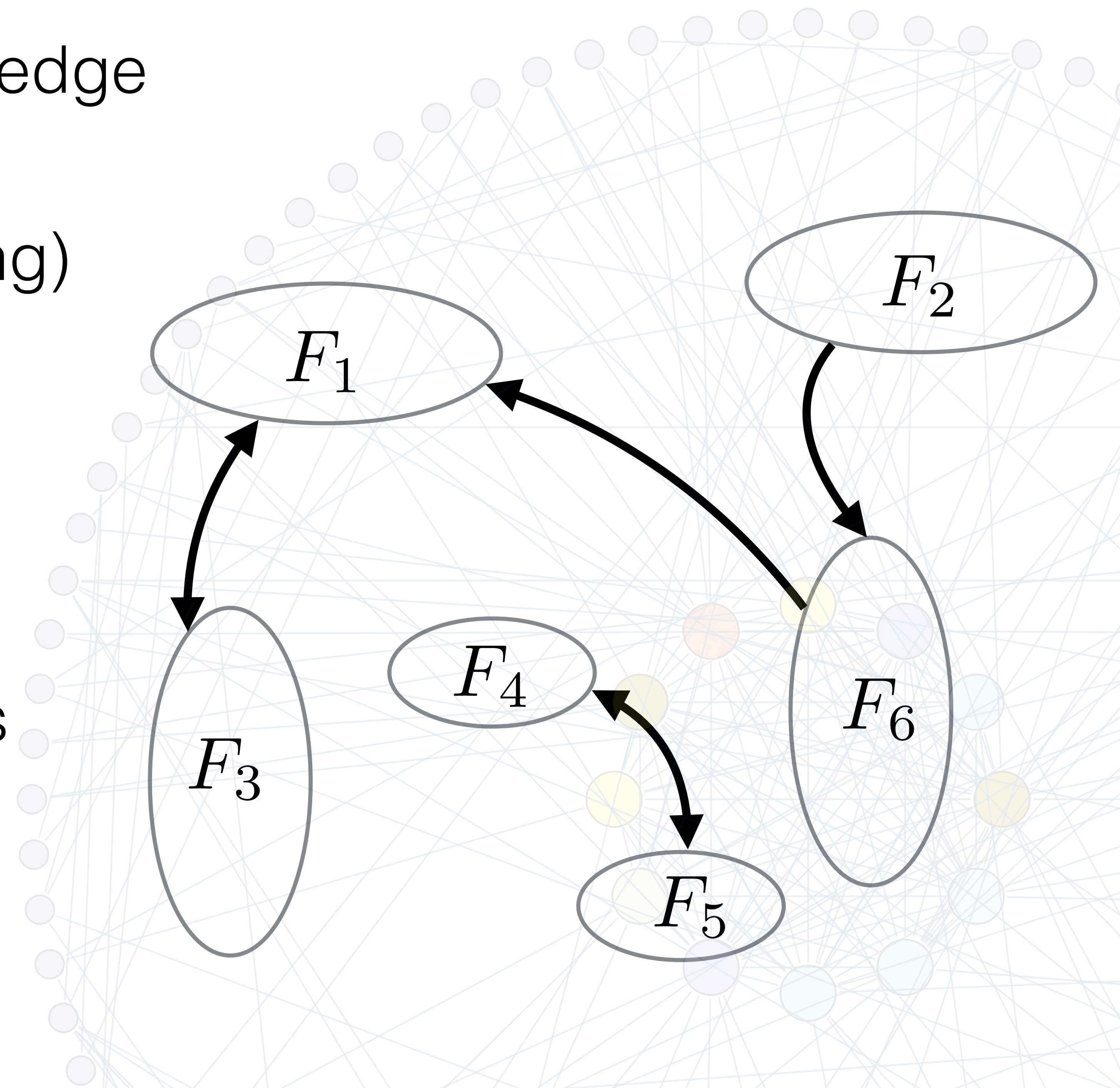
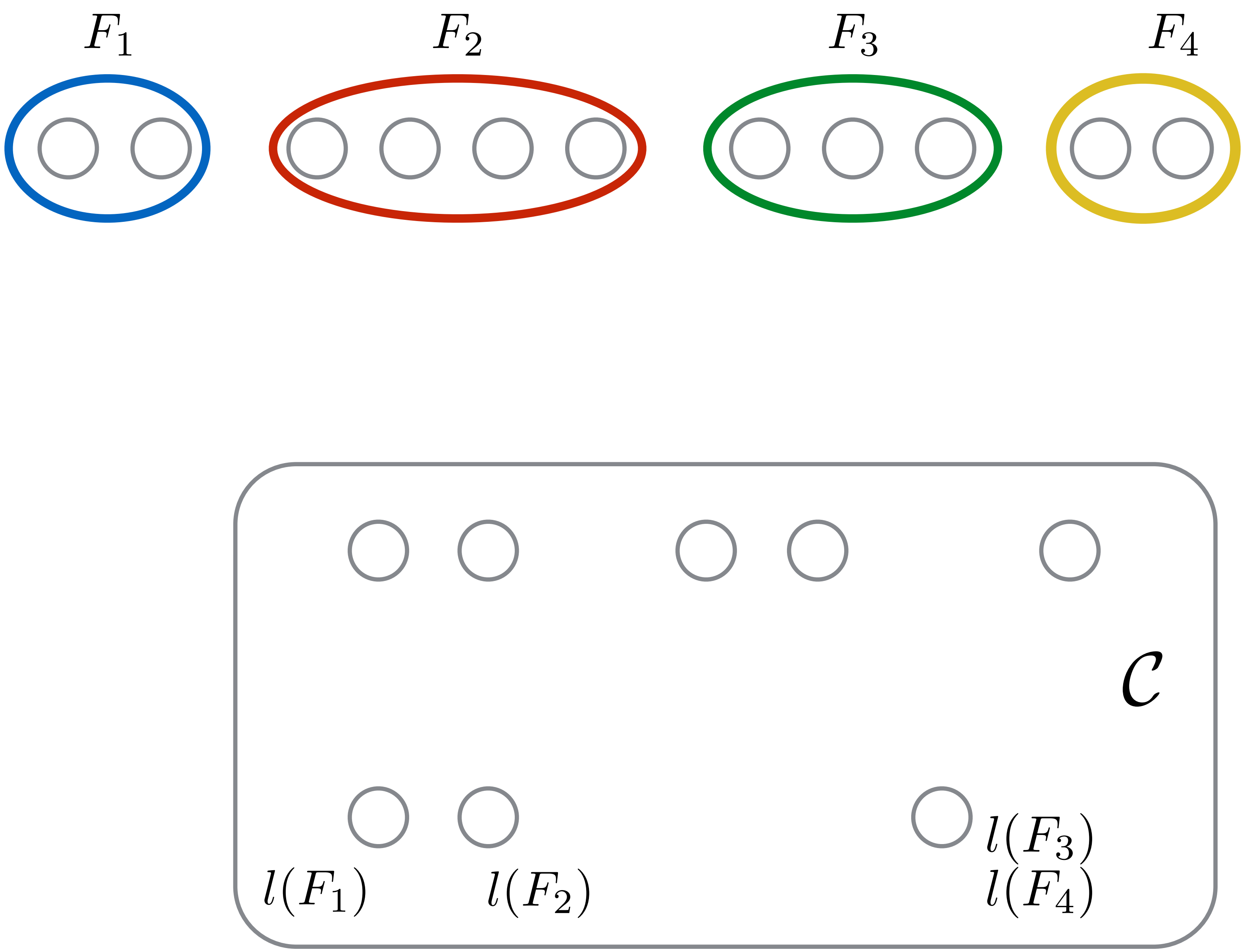


- We want:
  - each phase *O(log n)* rounds

Distributed MST for Core-Periphery Networks

Each node in ***V*** has two ***officials*** in ***C:***

1. Node's **Representative** - fixed
2. Fragment's **Leader** - may migrate in each phase



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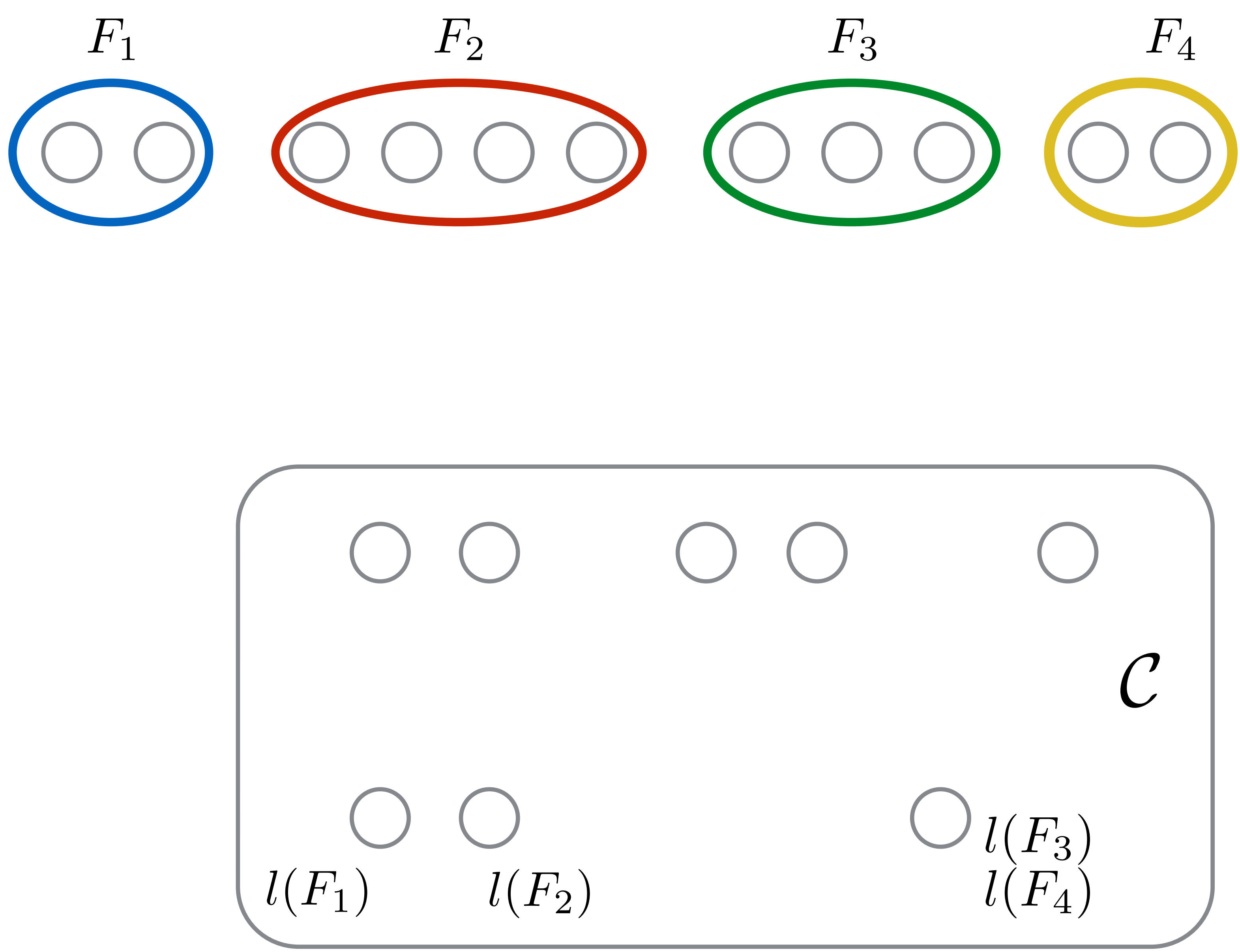

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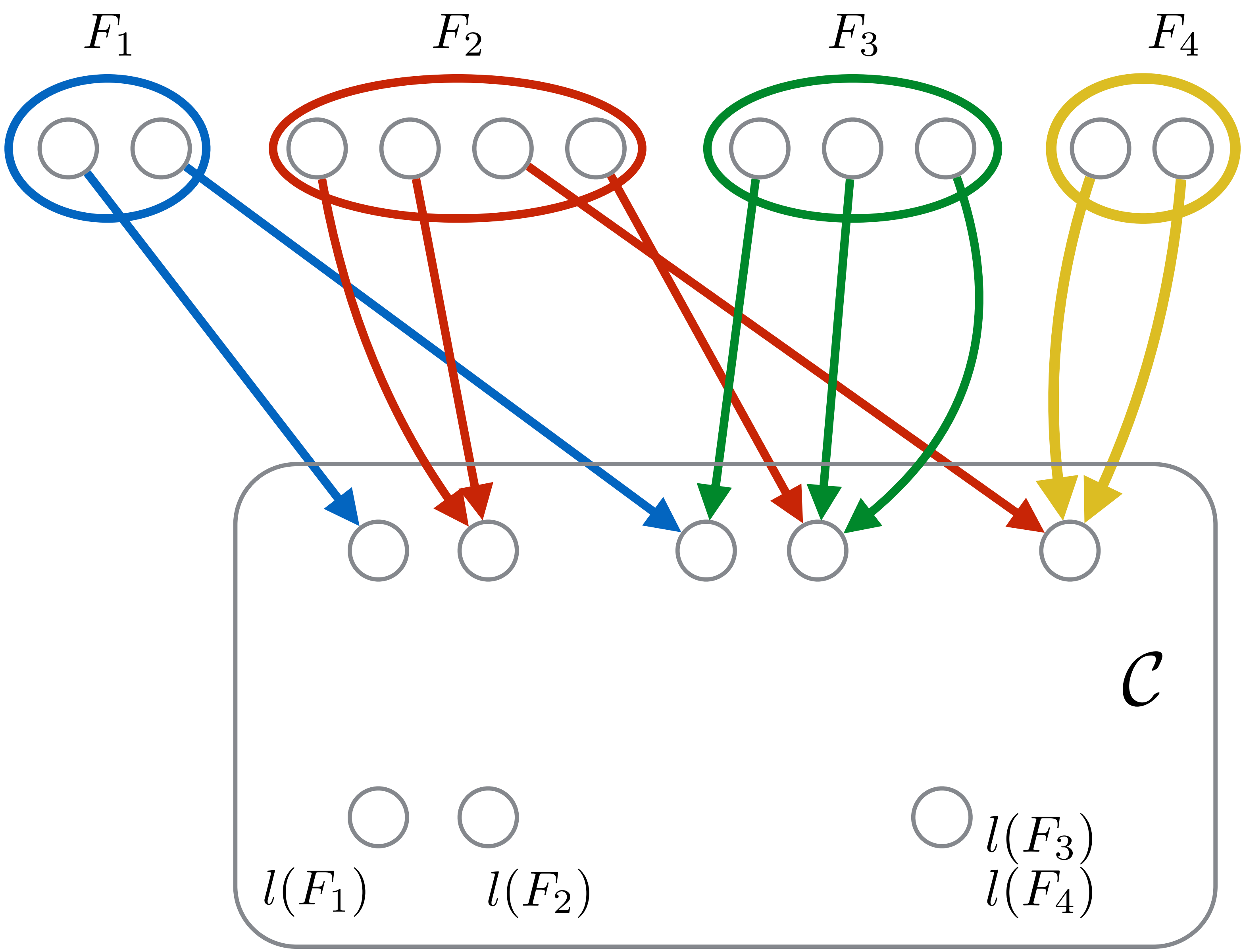

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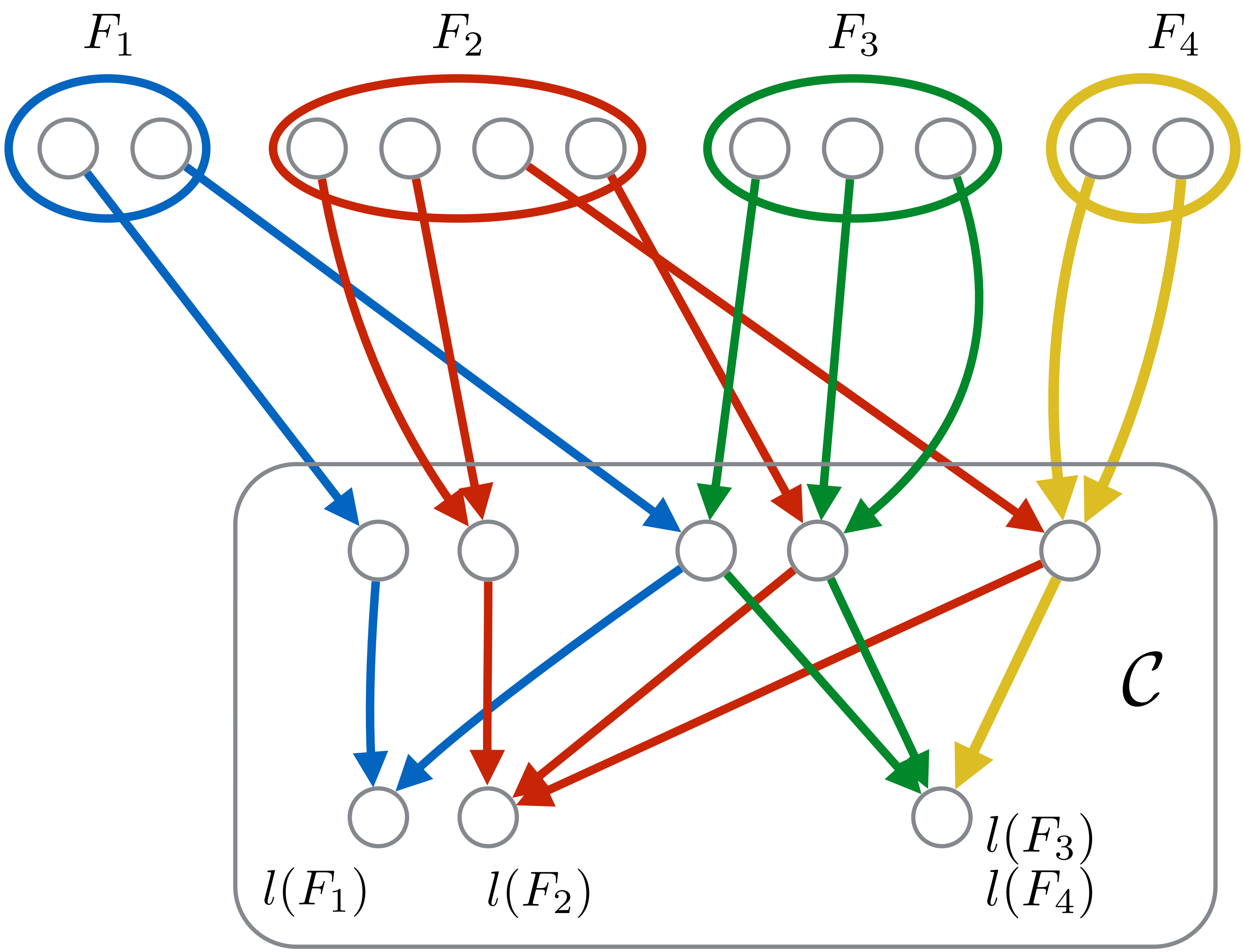

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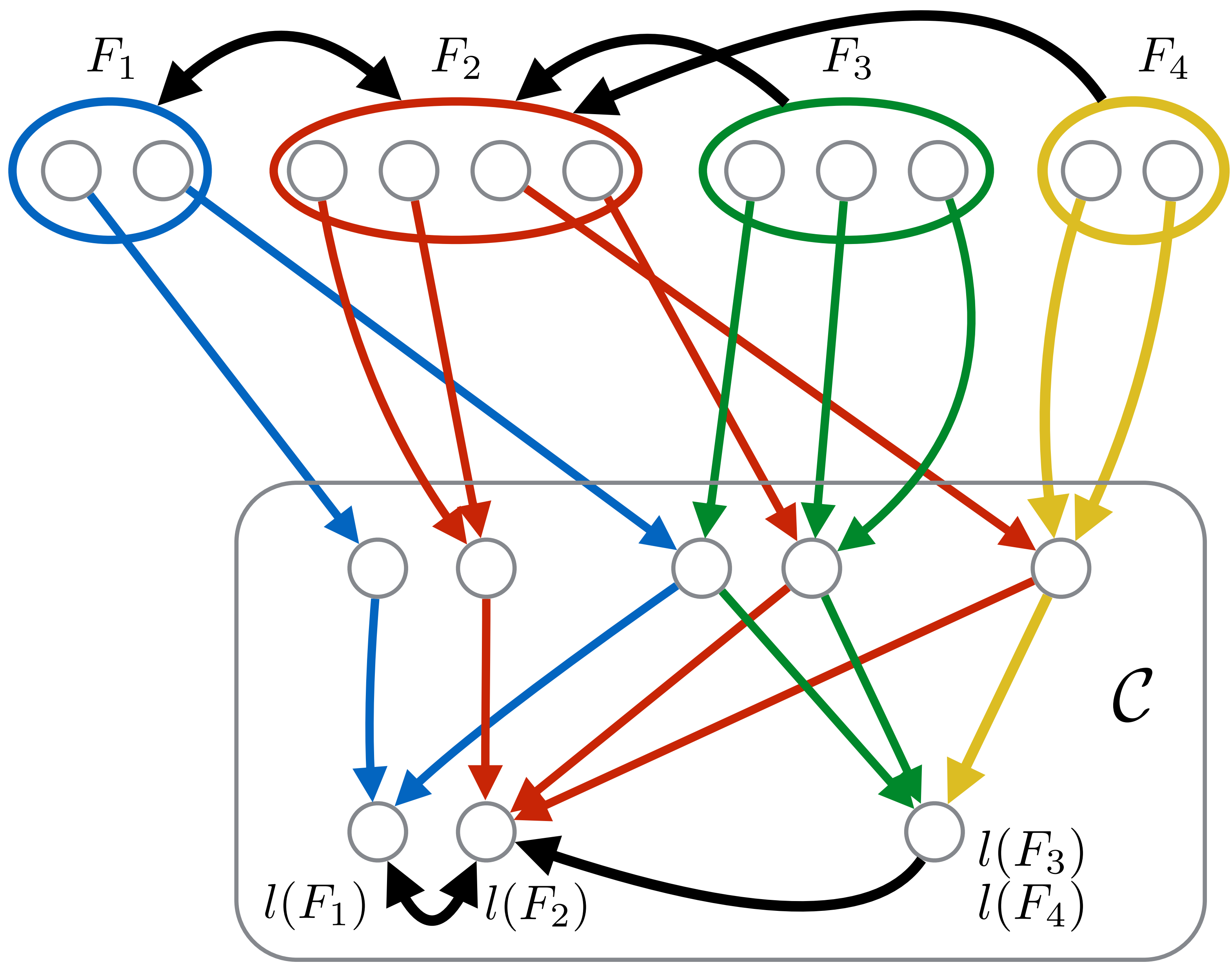
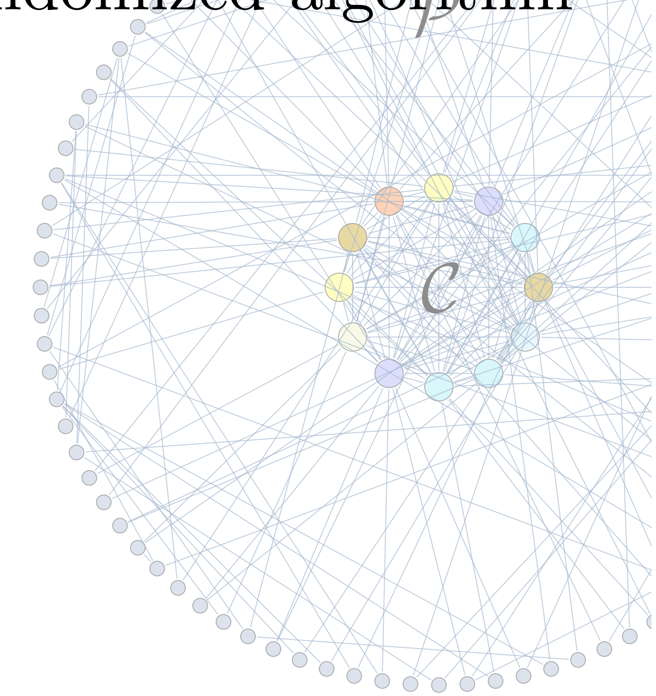
Leaders decide on merging



Algorithms for Core-Periphery Networks

Task	Running time on $\mathcal{CP}$ networks	Lower bounds	
		All Axioms	Any 2 Axioms
MST *	$O(\log^2 n)$	$\Omega(1)$	$\tilde{\Omega}(\sqrt[4]{n})$
Matrix transposition	$O(k)$	$\Omega(k)$	$\Omega(n)$
Vector by matrix multiplication	$O(k)$	$\Omega(k/\log n)$	$\Omega(n/\log n)$
Matrix multiplication	$O(k^2)$	$\Omega(k^2)$	$\Omega(n/\log n)$
Find my rank	$O(1)$	$\Omega(1)$	$\Omega(n)$
Find median	$O(1)$	$\Omega(1)$	$\Omega(\log n)$
Find mode	$O(1)$	$\Omega(1)$	$\Omega(n/\log n)$
Find number of distinct values	$O(1)$	$\Omega(1)$	$\Omega(n/\log n)$
Top r ranked by areas	$O(r)$	$\Omega(r)$	$\Omega(r\sqrt{n})$

k - maximum number of nonzero entries in a row or column. * - randomized algorithm



Summary

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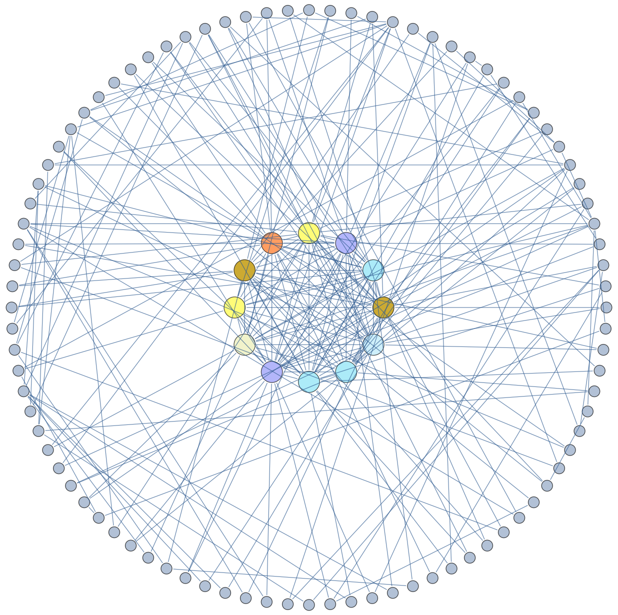
- Axiomatic approach
- Core-periphery is fast and cost efficient network architecture
- Simple algorithms design



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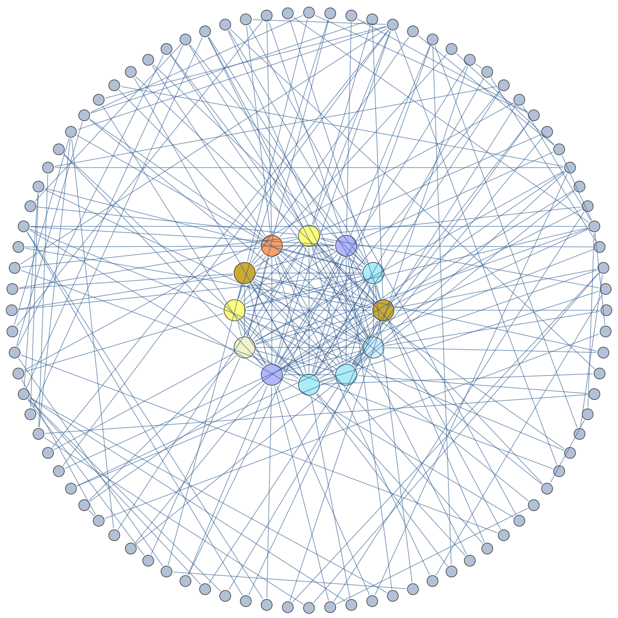

- Future work:
  - What else can be computed efficiently?
  - What are the basic building blocks in distributed computing?

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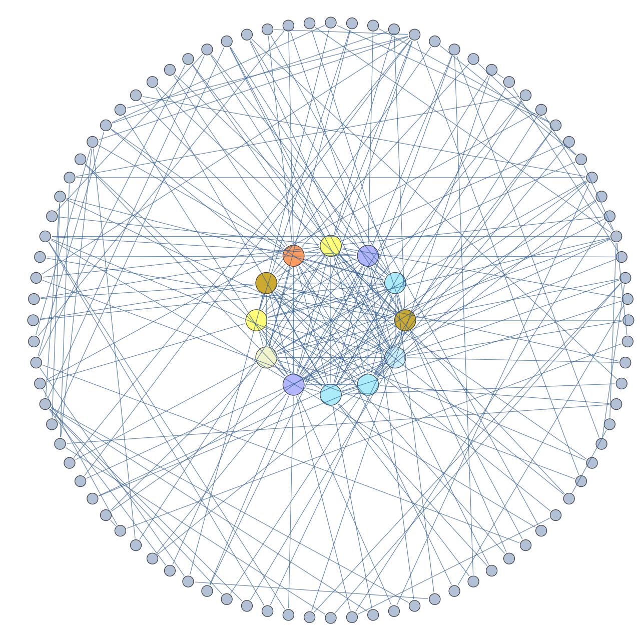

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Thank You!